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Numerical method for solving fractional coupled Burgers equations

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ABSTRACT

In this paper, we use the fractional variational iteration method (FVIM) to solve a time- and space-fractional coupled Burgers equations. Some numerical examples are presented to show the efficiency of considered method. A comparison of the proposed method is made with the exact solution, adomain decomposition method (ADM), generalized differential transformation method (GDTM) and homotopy perturbation method (HPM).

1. Introduction

In the lea of fractional differential equations and fractional calculus, the prevalent progress has been presumed in recent past. Prominent use of various projects which are mold by fractional order differential equations, lie in the lea of electromagnetic waves, ion-acoustic wave, bio-informatics, nano-technology, viscoelasticity, chemical engineering, mechanical engineering, electrode–electrolyte polarization, heat conduction, diffusion equations and almost every part of science and technology. A significant consideration has been given to approximate and exact solutions of differential equations involving fractional order derivative because of its eerie scope and applications in various fields. The solution techniques and their reliability are more important aspects than modeling dimensions of such type of differential equations. It is very essential to strut critic facts that procreate emergent divergence, bifurcation and convergence of the solutions of that model. To instate the aim of high accuracy and reliability of solutions, numerous approaches have been devised to find the solution of the differential equations having fractional order derivative. There are many neoteric methods that find analytic or numerical solutions of fractional differential equations like homotopy perturbation method, homotopy analysis method, finite volume method, adomain decomposition method and other various iterative methods. In 1997, He [1–4] developed a new technique, namely, variational iteration method (VIM) to solve linear and nonlinear differential equations.

In 2009, Odibat et al. [5] and Molliq et al. [6] applied VIM to solve fractional Zakharov–Kuznetsov equations. In 2011, Lu [7] and in 2012, 2015, Sakar et al. [8,9] applied VIM and AVIM to Fornberg–Whitham equation.

In this paper, coupled Burgers equations with time- and space-fractional derivatives of the form [10] is taken into consideration

$$\begin{cases} \dfrac{\partial^{\alpha_1} u}{\partial t^{\alpha_1}} = \dfrac{\partial^2 u}{\partial x^2} + 2u\dfrac{\partial^{\alpha_2} u}{\partial x^{\alpha_2}} - \dfrac{\partial (uv)}{\partial x}, \quad (0 < \alpha_i \le 1), \\ \dfrac{\partial^{\beta_1} v}{\partial t^{\beta_1}} = \dfrac{\partial^2 v}{\partial x^2} + 2v\dfrac{\partial^{\beta_2} v}{\partial x^{\beta_2}} - \dfrac{\partial (uv)}{\partial x}, \quad (0 < \beta_i \le 1), \end{cases} \tag{1}$$

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where in (1) α_1 and β_1 are order of time fractional derivative and α_2 and β_2 are order of space fractional derivative and these fulfil $0 < \alpha_i$, $\beta_i \le 1$ where $i = 1, 2$ and $t > 0$. Actually, we can form different response systems by changing at least one of the parameters. Eq. (1) can be converted into classical coupled Burgers equation by putting $\alpha_i = \beta_i = 1$.

It is important to study coupled Burgers equations for that the system (1) reduced to a simple model of sedimentation or evolution of scaled volume concentrations of two kinds of particles in fluid suspensions or colloids, under the effect of gravity [11]. Several approaches have been developed for this system [12,13] by many authors. Excellent work on coupled Burgers equation has been carried out by Dehghan et al. [14]. A generalized differential transformation and Homotopy perturbation methods for coupled Burger equation with time and space fractional derivative are proposed by Liu [15] and Yildirim [16]. However the coupled Burger equation with time and space fractional derivative by fractional variational iteration method has not been studied. The main aim of this paper is to develop an iterative method based on fractional variational iteration method to solve time- and space-fractional coupled Burgers equations.

2. Preliminaries

Definition 2.1. A real function $f(t)$, $t > 0$ is said to be in the space $C_\alpha, \alpha \in R$ if there exists a real number$p(> \alpha)$, such that $f(t) = t^p f_1(t)$ where $f_1 \in C[0, \infty]$. Clearly $C_\alpha \subset C_\beta$ if $\beta \le \alpha$ [17].

Definition 2.2. A function $f(t)$, $t > 0$is said to be in the space $C_\alpha^m, m \in N \cup \{0\}$, if $f^{(m)} \in C_\alpha$.

Definition 2.3. The left sided Riemann–Liouville fractional integral of order$\mu > 0$, [17–19] of a function $f \in C_\alpha, \alpha \ge -1$ is defined as:

$$I^\mu f(t) = \frac{1}{\Gamma(\mu)}\int_0^t \frac{f(\tau)}{(t-\tau)^{1-\mu}}d\tau = \frac{1}{\Gamma(\mu+1)}\int_0^t f(\tau)(d\tau)^\mu$$
$$I^0 f(t) = f(t).$$

Definition 2.4. The (left sided) Caputo fractional derivative of $f, f \in C_{-1}^m, m \in \mathrm{IN} \cup \{0\}$ [17–19],

$$D_t^\mu f(t) = \begin{cases} \left[I^{m-\mu} f^{(m)}(t)\right], & m-1 < \mu < m, m \in \mathrm{IN}, \\ \dfrac{d^m}{dt^m} f(t), & \mu = m \end{cases}$$

a. $I_t^\alpha f(x,t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1} f(x,s)ds, \alpha, t > 0$.
b. $D_t^\alpha u(x,t) = I_t^{m-\alpha}\frac{\partial^m u(x,t)}{\partial t^m}, m-1 < \alpha < m$.
c. $I^\mu t^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\mu+\gamma+1)} t^{\mu+\gamma}$.

3. The proposed FVIM method for the fractional coupled Burgers equation

To define solution process of (1) using fractional variational iteration method, we study the ensuing fractional differential equation

$$\begin{cases} \dfrac{\partial^{\alpha_1} u}{\partial t^{\alpha_1}} = \dfrac{\partial^2 u}{\partial x^2} + 2u\dfrac{\partial^{\alpha_2} u}{\partial x^{\alpha_2}} - \dfrac{\partial(uv)}{\partial x}, & (0 < \alpha_i \le 1), \\ \dfrac{\partial^{\beta_1} v}{\partial t^{\beta_1}} = \dfrac{\partial^2 v}{\partial x^2} + 2v\dfrac{\partial^{\beta_2} v}{\partial x^{\beta_2}} - \dfrac{\partial(uv)}{\partial x}, & (0 < \beta_i \le 1). \end{cases}$$

According to the FVIM, a correct functional [3] can be built for above equation as

$$u_{n+1}(x,t) = u_n + \frac{1}{\Gamma(1+\alpha_1)}\int_0^t \lambda_1\left(\frac{\partial^{\alpha_1} u_n}{\partial \tau^{\alpha_1}} - \frac{\partial^2 \tilde{u}_n}{\partial x^2} - 2\tilde{u}_n\frac{\partial^{\alpha_2}\tilde{u}_n}{\partial x^{\alpha_2}} - \frac{\partial(\tilde{u}_n\tilde{v}_n)}{\partial x}\right)(d\tau)^{\alpha_1}, \tag{2}$$

$$v_{n+1}(x,t) = v_n + \frac{1}{\Gamma(1+\beta_1)}\int_0^t \lambda_2\left(\frac{\partial^{\beta_1} v_n}{\partial \tau^{\beta_1}} - \frac{\partial^2 \tilde{v}_n}{\partial x^2} - 2\tilde{v}_n\frac{\partial^{\beta_2}\tilde{v}_n}{\partial x^{\beta_2}} - \frac{\partial(\tilde{u}_n\tilde{v}_n)}{\partial x}\right)(d\tau)^{\beta_1}. \tag{3}$$

Now by the variational theory λ_1, λ_2 must satisfy $\frac{\partial^{\alpha_1}\lambda_1}{\partial \tau^{\alpha_1}} = 0$ and $1 + \lambda_1|_{\tau=t} = 0$ and $\frac{\partial^{\beta_1}\lambda_1}{\partial \tau^{\beta_1}} = 0$ and $1 + \lambda_2|_{\tau=t} = 0$. From these equations, we obtain $\lambda_1, \lambda_2 = -1$ and a new correction functional

$$u_{n+1}(x,t) = u_n - \frac{1}{\Gamma(1+\alpha_1)}\int_0^t \left(\frac{\partial^{\alpha_1} u_n}{\partial \tau^{\alpha_1}} - \frac{\partial^2 u_n}{\partial x^2} - 2u_n\frac{\partial^{\alpha_2} u_n}{\partial x^{\alpha_2}} - \frac{\partial(u_n v_n)}{\partial x}\right)(d\tau)^{\alpha_1}, \tag{4}$$

$$v_{n+1}(x,t) = v_n - \frac{1}{\Gamma(1+\beta_1)}\int_0^t \left(\frac{\partial^{\beta_1} v_n}{\partial \tau^{\beta_1}} - \frac{\partial^2 v_n}{\partial x^2} - 2v_n(x,\tau)\frac{\partial^{\beta_2} v_n}{\partial x^{\beta_2}} - \frac{\partial(u_n v_n)}{\partial x}\right)(d\tau)^{\beta_1}. \tag{5}$$