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Numerical solution of a fuzzy stochastic single-species age-structure model in a polluted environment

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ABSTRACT

This paper presents an investigation of a fuzzy stochastic single-species age-structure model in a polluted environment. Both the fuzziness of the initial condition and the stochastic disturbance of the environment are incorporated into the model. By using the theory of fuzzy stochastic differential equation (FSDE) and the successive approximation, the global existence and uniqueness of solutions of the model are proved. In addition, the error estimation and stability of the numerical solutions are obtained. Furthermore, making use of Euler–Maruyama (EM) method, the convergence of the EM numerical approximation is established. Numerical simulations are carried out to verify the theoretical results. Our results show that the technique of numerical solution of FSDE can be used to estimate the evolution tendency of the population density in a polluted environment.

1. Introduction

With the fast development of the industry and agriculture, there is no denying the fact that environmental pollution, which has caused many serious ecological problems (e.g. the decreasing bio-diversity, the verge of extinction of some species, etc.), is a widespread phenomenon presented all over the world [1,2]. Thus, it is vital to regulate toxicant suitable and to estimate the risk of the species in a polluted environment. Recently, various deterministic models have been proposed to study the effect of toxicant [3–9]. In particular, Luo and He [9] established a toxicant-population model with age-structure in a polluted environment which takes the form

$$\begin{cases} \dfrac{\partial P(a,t)}{\partial a} + \dfrac{\partial P(a,t)}{\partial t} = -\mu(a,t,C_0(t))P(a,t), & \text{in } Q \\ \dfrac{dC_0(t)}{dt} = kC_e(t) - (l+m)C_0(t), & \text{in } [0,T] \\ \dfrac{dC_e(t)}{dt} = -k_1C_e(t)x(t) + l_1C_0(t)x(t) - hC_e(t) + u(t), & \text{in } [0,T] \\ P(0,t) = \displaystyle\int_0^A \beta(a,t,C_0(t))P(a,t)da, & \text{in } [0,T] \\ P(a,0) = P_0(a), & \text{in } [0,A] \\ 0 \le C_0(0) \le 1, \quad 0 \le C_e(0) \le 1, & \\ x(t) = \displaystyle\int_0^A P(a,t)da, & \text{in } [0,T] \end{cases} \tag{1.1}$$

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Table 1
The biological meanings of the parameters in model (1.1).

Parameters	Biological meanings
$\mu(a, t, C_0(t))$	The mortality rate function of the population of age a at time t
$\beta(a, t, C_0(t))$	The fertility rate function of the population of age a at time t
$u(t)$	The exogenous total toxicant input into environment at time t
$A(0 < A < \infty)$	The maximum life expectancy
k	The net organismal uptake rate of toxicant from the environment
l	The net organismal excretion rate of toxicant
m	Depuration rate of toxicant due to metabolic process and other losses
h	The total loss rate of the toxicant from the environment
k_1	The rate of toxicant taken from the environment by the population
l_1	The rate of toxicant input to the environment from the population

where $Q = [0, A] \times [0, T]$, and $P(a, t)$ denotes the density of the population of age a at time t, $C_0(t)$ is the concentration of toxicant in the organism at time t, $C_e(t)$ is the concentration of toxicant in the environment at time t, $x(t)$ is the total density of the population at time t. The biological meanings of all the parameters in model (1.1) are listed in Table 1. For this model, the authors obtained the existence and uniqueness of nonnegative solution, and verified the existence of optimal control policy by means of Ekeland's variational principle.

As well known, the species are inevitably affected by stochastic environmental fluctuation, which is an important component in realism. May [10] pointed out the fact that due to environmental fluctuation, the birth rates, carrying capacity, competition coefficients and other parameters involved in the system exhibit random fluctuation to a greater or lesser extent. Therefore, the study of polluted population models with stochastic perturbance have recently been a topic of considerable interest [11,12]. On the other hand, due to the lack of information, errors in the measurement process and determining the initial conditions, the knowledge about biological parameters and/or the initial conditions are often incomplete or vague [13–16]. If these factors are further taken into consideration in model (1.1), we obtain the following fuzzy stochastic model:

$$\begin{cases} \dfrac{\partial P(a,t)}{\partial a} + \dfrac{\partial P(a,t)}{\partial t} = -\mu(a, t, C_0(t))P(a,t) + \langle g(t, P(a,t))\dfrac{dB(t)}{dt}\rangle, & \text{in } Q \\ \dfrac{dC_0(t)}{dt} = kC_e(t) - (l+m)C_0(t), & \text{in } [0,T] \\ \dfrac{dC_e(t)}{dt} = -hC_e(t) + u(t), & \text{in } [0,T] \\ P(0,t) = \displaystyle\int_0^A \beta(a, t, C_0(t))P(a,t)da, & \text{in } [0,T] \\ P(a,0) = P_0(a), & \text{in } [0,A] \\ 0 \le C_0(0) \le 1, \quad 0 \le C_e(0) \le 1, & \\ x(t) = \displaystyle\int_0^A P(a,t)da, & \text{in } [0,T] \end{cases} \tag{1.2}$$

where $g(t, P(a, t))$ denotes the diffusion coefficient, which dependents on a, t and P. $\{B(t)\}_{t\in I}$ is one-dimensional $\{A(t)\}_{t\in I}$-adapted Brownian motion. $\langle V\rangle$ denotes the image of V by the embedding $\langle\cdot\rangle : V \to \mathcal{F}(V)$. In the sense of fuzzy stochastic differential equation, $g : (Q \times \Omega) \times \mathcal{F}(V) \to V$, and $P_0(a) \in \mathcal{L}^2(Q \times \Omega, \mathcal{N}; \mathcal{F}(V))$ are fuzzy random variables. Other parameters have the similar biological meanings as in model (1.1). To the best of our knowledge, up to now there are few investigations about the fuzzy stochastic age-structure population models in a polluted environment. In this paper, by considering the initial populations density $P_0(a)$ to be fuzzy stochastic variables, we will devote our main attention to the investigation on the tendency range of the population density under the disturbance of white noise and pollution.

Recently, the research on the fuzzy stochastic differential equation (FSDE) has attracted many authors' attention. For example, under a boundedness and the Lipschitz condition, Malinowski [17] obtained the existence and uniqueness of solution to fuzzy stochastic differential equations; Malinowski [18] respectively studied the Picard type, the Caratheodory type and Maruyama type approximate solutions for fuzzy stochastic integral equations. Readers may refer to [19–25] and the references therein for related studies on this respect. Notice also that FSDE cannot be solved explicitly, numerical solution methods have become important and essential in the study of FSDE. In this paper, we will use the related theory and numerical solution methods of FSDE to deal with model (1.2). We will prove the existence and uniqueness of the solution, and obtain the error estimation and stability of the numerical solutions with respect to initial values. Moreover, the convergence of the Euler–Maruyama (EM) numerical approximation is established.

This paper is organized as follows. Some related preliminaries are presented in the next section. In Section 3, by using of the theory of fuzzy stochastic differential equation and the successive approximation, the global existence and uniqueness of solutions of the model are proved, and the error estimation and stability of the numerical solutions are obtained. The convergence