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Role of additional food in eco-epidemiological system with disease in the prey

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ABSTRACT

An eco-epidemiological system with disease in the prey incorporating additional food to predator is proposed. The main objective of this study is to show the role of additional food in an eco-epidemiological system. We analyze the proposed system by calculating two reproduction numbers. The dynamical behavior of the system is investigated from the point of view of stability and persistence both analytically and numerically. Using Pontryagin's Maximum Principle, an optimal control problem is formulated and solved in presence of additional food to achieve the control of disease. Numerical results illustrate that there exists a critical infection rate above which disease free system can not be reached in absence of additional food. On the other hand suitable additional food has the capability to obtain a disease free system up to certain infection level. The system becomes disease free also in presence of seasonally varying infection rate providing suitable additional food to predator. This study introduces a new non-chemical method for controlling disease in eco-epidemiological system.

1. Introduction

Eco-epidemiology is important in the understanding of disease and disease emergence. It is the field of study at the confluence of ecology, which studies population dynamics, epidemiology, and infectious diseases in biological communities. Therefore, it is essential to study the effects of epidemiological features on real ecological populations. Mathematical study of such eco-epidemiological models explored various unknown aspects of ecological populations. It is basically based on some specific properties of population growth, the spread rules of infectious diseases, and the related social factors to construct mathematical models [1–3]. Recently, Szolnoki et al. [4] investigated well-mixed predator–prey systems applying evolutionary games to maximize the chances of populations survival. Basically, it is based on the dynamic properties of infectious diseases, to analyze the dynamical behavior and to do some simulations [5–7]. The research results are helpful to determine the key factors of the spread of infectious disease and to seek the optimum strategies for controlling the spread of infectious diseases. Mathematical modeling studies on disease dominated ecological systems have addressed issues like disease related mortality, reduction in reproduction, change in population sizes and disease induced oscillation of population states [8]. The disease factor in predator–prey system was first introduced by Anderson and May [9] and they investigated the key factor to destabilize the predator–prey interactions. Starting from some famous mathematical models [10–12], so many numbers of epidemic models were developed [13,14].

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There has been a growing interest in the study of disease control factors in eco-epidemiological systems. The investigation of disease control techniques or mechanisms are one of the main success for studying the epidemic models. There are so many techniques and methods for disease control of a system using chemical treatments such as vaccination, isolation and insecticide. Recently, Zhao et al. [15] and Fukuda et al. [16] investigated a disease control mechanism in multiplex networks and social networks of populations through the applications of vaccination. But, most of the techniques of disease control in eco-epidemiological systems are the use of chemical. These chemical use in ecological systems cause serious problems like eradication of lower species, pollutions, etc in real world system. So, it is important to use alternative technique of chemical use. The application of alternative non-chemical methods of disease control continues to gain significance and popularities. Use of Additional food is one of most essential and new important non-chemical treatment for disease control. Some investigations [17–22] were done for disease control in epidemic models providing additional food to predator. Recently, Sahoo and Poria [20–22] proposed a disease induced predator–prey model providing additional food to predator to obtain disease free system. They investigated the effects of additional food on epidemic models with infection rate of simple mass action kinetics. So, it is interesting to investigate the role of additional food on epidemic model with saturation incidence kinetics. This is totally a new research area which has applications in many important practical fields. This paper aims to analyze the role of additional food for controlling disease in the system. The paper is organized as follows: In Section 2, the model is formulated. The dynamical behavior of two subsystems are discussed in Section 3. Section 4 contains the dynamical behaviors of full system with global stability and persistence conditions. Optimal control to disease in presence of additional food is described in Section 5. Numerical results of the subsystems and full system are described in Section 6. Finally, conclusions is drawn in Section 7.

2. Model formulation

The predator–prey model with disease in the prey providing additional food to predator is formulated under the following assumptions:

(a) In presence of disease, the prey population consists of two subclasses, namely, susceptible prey $S(t)$ and infected prey $I(t)$ and the density of the predator is denoted by $P(t)$ at any time t.
(b) In absence of disease, the susceptible prey population grows logistically with intrinsic growth rate R_0 and environmental carrying capacity K_0.
(c) It is assumed that the disease is spread only among the prey population and the disease is not genetically inherited. The infected population does not recover or become immune.
(d) Susceptible prey becomes infected when they come in contact with infected prey. The contact process is assumed to follow the saturation incidence kinetics, with W_1 measuring the force of infection, E_1 the saturation constant and W_2 the inhibition effect. This incidence rate is more realistic than the bilinear or simple mass action kinetics, as it includes the behavioral change and crowding effect of the infected individuals and also prevents unboundedness of contact rate [23–25].
(e) Predators are provided with additional food of constant biomass A which is distributed uniformly in the habitat. The constant biomass assumption is valid for many arthropod predators because they can feed on plant-provided additional food sources such as pollen or nectar which approximately remains constant [21]. Almost all predators will attempt to switch to another prey when the preferred prey is in low numbers and they may also resort to scavenging or a herbivorous diet if possible [26].
(f) The number of encounters per predator with additional food is proportional to the density of the additional food.
(g) The proportionality constant characterizes the ability of the predator to identify the additional food.

Let us assume that h_1 (h_2), e_1 (e_2), n_1 (n_2) respectively represent the handling time of the predator per unit quantity of susceptible prey (infected prey), ability of the predator to detect the susceptible prey (infected prey) and the nutritional value of the susceptible prey (infected prey) [27]. We also consider that h_3, e_3, n_3 respectively represent the handling time of the predator per unit quantity of additional food, ability of the predator to identify the additional food and the nutritional value of the additional food. Therefore, using above assumptions, the predator–prey model is of the following form

$$\begin{aligned}
\frac{dS}{dT} &= R_0 S\left(1-\frac{S+I}{K_0}\right) - \frac{W_1 SI}{E_1+W_2 I} - \frac{e_1 SP}{1+e_3h_3A+e_1h_1S},\\
\frac{dI}{dT} &= \frac{W_1 SI}{E_1+W_2 I} - \frac{e_2 IP}{1+e_3h_3A+e_2h_2I} - D_1 I,\\
\frac{dP}{dT} &= \frac{(n_1e_1S+n_3e_3A)P}{1+e_3h_3A+e_1h_1S} + \frac{(n_2e_2I+n_3e_3A)P}{1+e_3h_3A+e_2h_2I} - D_2 P.
\end{aligned} \tag{1}$$

The model (1) can be written in the form