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Convolutions of the generalized Morgan–Voyce polynomials

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ABSTRACT

We consider the following classes of polynomials: $B^{(s)}_{n,m}(x), C^{(s)}_{n,m}(x), b^{(s)}_{n,m}(x)$ and $c^{(s)}_{n,m}(x)$, which are convolutions of the generalized Morgan–Voyce polynomials, where s is a nonnegative integer. These convolutions are related to the classical Morgan–Voyce, Chebyshev and Jacobsthal polynomials.

1. Introduction

Classical Morgan–Voyce polynomials are the well–known polynomials. There are many classes of polynomials which are related to the Morgan–Voyce polynomials. In this note we are motivated by some recent papers in this topic, such as [1,4,7,8].

First, we define convolutions $B^{(s)}_{n,m}(x), C^{(s)}_{n,m}(x), b^{(s)}_{n,m}(x)$ and $c^{(s)}_{n,m}(x)$. Then, we prove some properties of new polynomials. Finally, we define and we consider the convolution polynomials for the generalized Chebyshev polynomials $\Omega_{n,m}(x)$ and $V_{n,m}(x)$.

Throughout this paper we use $\mathbb{N}$ to denote the set of all nonnegative integers.

The generalized Morgan–Voyce polynomials are given by corresponding generating functions (see [1, 4]:)

$$\sum_{n=1}^{\infty} B_{n,m}(x)t^{n-1} = (1-(2+x)t+t^m)^{-1},\ (B_{0,m}(x)=0), \tag{1.1}$$

$$\sum_{n=0}^{\infty} C_{n,m}(x)t^{n} = \left(2-(2+x)t^{m-1}\right)(1-(2+x)t+t^m)^{-1}, \tag{1.2}$$

$$\sum_{n=1}^{\infty} b_{n-1,m}(x)t^{n-1} = \left(1-(1+x)t^{m-1}\right)(1-(2+x)t+t^m)^{-1}, \tag{1.3}$$

$$\sum_{n=0}^{\infty} c_{n,m}(x)t^{n} = \left(-1+(3+x)t^{m-1}\right)(1-(2+x)t+t^m)^{-1}, \tag{1.4}$$

where m is a positive integer.

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The main purpose of this paper is to introduce and to investigate the polynomials $B^{(s)}_{n,m}(x), C^{(s)}_{n,m}(x), b^{(s)}_{n,m}(x)$ and $c^{(s)}_{n,m}(x)$, where $s \in \mathbb{N}$.

Basic properties of these polynomials are developed in [1, 4, 5, 8, 9].

2. Convolution for $B_{n,m}(x)$

The sth convolution polynomials $B^{(s)}_{n,m}(x)$ of the polynomials $B_{n,m}(x)$ are defined as

$$\sum_{n=1}^{\infty} B^{(s)}_{n,m}(x)t^{n-1} = (1-(2+x)t+t^m)^{-(s+1)}, \tag{2.1}$$

where $B^{(0)}_{n,m}(x) = B_{n,m}(x)$.

Notice that the polynomials $B_{n,m}(x)$ are the special case of the Humbert polynomials $P_n(m,x,y,p,C)$, i.e., the next relation holds ([11], see also [6])

$$B_{n,m}(x) = P_{n-1}(m,(x+2)/m,1,-(s+1),1).$$

Namely, Humbert polynomials are given as

$$(C-mxt+yt^m)^p = \sum_{n=0}^{\infty} P_n(m,x,y,p,C)t^n,$$

where m is positive integer and the other parameters are unrestricted in general.

Next, from (2.1), using the known method, we find that the next relation

$$B^{(s)}_{n,m}(x) = \sum_{k=0}^{[(n-1)/m]} \frac{(-1)^k (s+1)_{n-1-(m-1)k}}{k!(n-1-mk)!}(2+x)^{n-1-mk} \tag{2.2}$$

is an explicit formula of the polynomials $B^{(s)}_{n,m}(x)$.

Remark 1. Using the known relations (see [4]), the explicit formula (2.2) can be written in the following form, for $n := n+1$:

$$B^{(s)}_{n+1,m}(x) = \frac{2^n(s+1)_n}{n!}\, {}_mF_{m-1}\left[\begin{matrix} \frac{-n}{m}, \frac{1-n-m}{m}, \ldots, \frac{m-1-n}{m}; \frac{(m/(2+x))^m}{(m-1)^{m-1}} \\ \frac{-s-n}{m-1}, \frac{1-s-n}{m-1}, \ldots, \frac{m-2-s-n}{m-1}; \end{matrix}\right]$$

where

$${}_mF_{m-1}\left[\begin{matrix} a_1, a_2, \ldots, a_m; z \\ b_1, b_2, \ldots, b_{m-1}; \end{matrix}\right] = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n \cdots (a_m)_n}{(b_1)_n (b_2)_n \cdots (b_{m-1})_n} \cdot \frac{z^n}{n!}$$

is the generalized hypergeometric function ([11,12]).

Example 1. For $s = 0,1,2$ and $m = 3$, we get some initial members of $B^{(s)}_{n,3}(x)$ and some initial members of numbers $B^{(s)}_{n,3}(1) = B^{(s)}_{n,3}$. These sequences are given by Table 1 and Table 2, respectively.

Now, differentiating (2.1), with respect to t, we get:

$$\sum_{n=1}^{\infty} (n-1)B^{(s)}_{n,m}(x)t^{n-2} = -(s+1)(1-(2+x)t+t^m)^{-(s+2)}\left(-(2+x)+mt^{m-1}\right).$$

Hence, we find that the following recurrence relation holds

$$(n-1)B^{(s)}_{n,m}(x) = (s+1)(2+x)B^{(s+1)}_{n-1,m}(x) - m(s+1)B^{(s+1)}_{n-m,m}(x). \tag{2.3}$$

Next, we are going to prove the following theorem.

Theorem 1. *For all $n \geqslant m$ $(n,s \in \mathbb{N})$, $m \geqslant 1$, it holds*

$$B^{(s)}_{n,m}(x) = B^{(s+1)}_{n,m}(x) - (2+x)B^{(s+1)}_{n-1,m}(x) + B^{(s+1)}_{n-m,m}(x). \tag{2.4}$$