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A convergent least-squares regularized blind deconvolution approach

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ABSTRACT

The aim of this work is to present a new and efficient optimization method for the solution of blind deconvolution problems with data corrupted by Gaussian noise, which can be reformulated as a constrained minimization problem whose unknowns are the point spread function (PSF) of the acquisition system and the true image. The objective function we consider is the weighted sum of the least-squares fit-to-data discrepancy and possible regularization terms accounting for specific features to be preserved in both the image and the PSF. The solution of the corresponding minimization problem is addressed by means of a proximal alternating linearized minimization (PALM) algorithm, in which the updating procedure is made up of one step of a gradient projection method along the arc and the choice of the parameter identifying the steplength in the descent direction is performed automatically by exploiting the optimality conditions of the problem. The resulting approach is a particular case of a general scheme whose convergence to stationary points of the constrained minimization problem has been recently proved. The effectiveness of the iterative method is validated in several numerical simulations in image reconstruction problems.

1. Introduction

Image deconvolution is an extremely prolific field which on one hand finds applications in a large variety of areas (physics, medicine, engineering,...) and on the other hand rounds up the efforts of a large community of mathematicians working on inverse problems and optimization methods. Most of the resulting works deal with the ill-conditioned discrete problem in which the blurring matrix is assumed to be known and the goal is to find a good approximation of the unknown image by means of some regularization approaches [1]. However, in many real applications the blurring matrix is not completely known due to a lack of information on the acquisition model and/or to external agents which corrupt the measured image (atmospheric turbulence, thermal blooming,...). This situation is known as *blind deconvolution* and most strategies to approach this problem are based on a simultaneous recovery of both the image approximation and the point spread function (PSF) of the acquisition system. Blind deconvolution is a very actual field and a much more challenging problem than the image deconvolution one, due to the strongly ill-posedness caused by the non-uniqueness of the solution. A review of some recent results can be found e.g. in two recent papers by Almeida & Figuereido and Oliveira et al. [2,3], even if many other different approaches have been developed. If the noise corrupting the measured data is assumed to have a Gaussian nature,

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blind deconvolution can be addressed by means of the constrained minimization of the squared Euclidean norm of the residuals plus some regularization terms suitably chosen according to the object and PSF to be reconstructed. An alternative formulation involves the unconstrained minimization of the same objective function with the addition of the indicator functions of the feasible sets, which can be numerically solved with forward–backward splitting methods [4,5].

The starting point of this paper is a proximal alternating method recently proposed by Bolte et al. [6] for a more general class of nonconvex and nonsmooth minimization problems, in which the parameters defining the method are fixed by using the Lipschitz constants of the partial gradients of the least-squares plus regularization part of the objective function. Convergence of the resulting sequence to a stationary point is ensured by the Kurdyka–Łojasiewicz property [7–9]. In particular, for the specific case of blind deconvolution, we extend the convergence results proved in [6] to a wider range of parameters, which allows the corresponding scheme to converge much faster toward the limit point. Moreover, we introduce a practical adaptive choice of the parameters based on a measure of the optimality condition violation, and we test the proposed algorithm in some simulated numerical experiments.

The paper is organized as follows: in Section 2 we introduce the blind deconvolution from Gaussian data and we recall the specific formulation of the proximal alternating linearized minimization proposed in [6] for this problem. Our proposed extension of the scheme is described in Section 3, together with the analogous convergence results and some hints for the choice of the parameters. Section 4 is devoted to some numerical experiments on synthetic datasets, while our conclusions are given in Section 5.

2. Problem setting

When dealing with Gaussian noise, blind deconvolution can be modeled as the minimization problem

$$\min_{\boldsymbol{x}\in X,\boldsymbol{h}\in H} F(\boldsymbol{x},\boldsymbol{h}) = \frac{1}{2}\|\boldsymbol{h}*\boldsymbol{x}-\boldsymbol{g}\|^2 + \lambda_1 R_1(\boldsymbol{x}) + \lambda_2 R_2(\boldsymbol{h}), \tag{1}$$

where $X \subseteq \mathbb{R}^n$ and $H \subseteq \mathbb{R}^m$ are the nonempty, closed and convex feasible sets of the unknown image $\boldsymbol{x}$ and PSF $\boldsymbol{h}$, respectively, $*$ denotes the convolution operator, $\boldsymbol{g}$ is the measured image, $R_1,\ R_2$ are differentiable regularization terms and $\lambda_1,\ \lambda_2$ are positive regularization parameters. As usual, the computation of the convolution product is performed by means of a matrix–vector multiplication $\boldsymbol{h}*\boldsymbol{x} = \boldsymbol{H}\boldsymbol{x} = \boldsymbol{X}\boldsymbol{h}$, where $\boldsymbol{X}$ and $\boldsymbol{H}$ are suitable structured matrices depending on the choice of the boundary conditions [10]. Examples of regularization terms frequently used in the applications are the following:

- $R^{\mathrm{T0}}(\boldsymbol{z}) = \|\boldsymbol{z}\|^2$ (Tikhonov regularization of order 0);
- $R^{\mathrm{T1}}(\boldsymbol{z}) = \|\nabla\boldsymbol{z}\|^2$ (Tikhonov regularization of order 1);
- $R^{\mathrm{HS}}(\boldsymbol{z}) = \sum_i \sqrt{\|(\nabla\boldsymbol{z})_i\|^2+\beta^2}$ (hypersurface regularization), where $(\nabla\boldsymbol{z})_i$ is the 2-vector representing the discretization of the gradient of $\boldsymbol{z}$ at pixel i in the horizontal and vertical directions.

As concerns the feasible sets, we consider non-negative images and non-negative and normalized PSFs, therefore we assume

$$X = \{\boldsymbol{x}\in\mathbb{R}^n | \boldsymbol{x} \geqslant \boldsymbol{0}\},$$
$$H = \left\{\boldsymbol{h}\in\mathbb{R}^m | \boldsymbol{h}\geqslant\boldsymbol{0}, \sum_i h_i = 1\right\}.$$

If we denote with $\mathcal{I}_X$ and $\mathcal{I}_H$ the indicator functions of X and H, respectively, then problem (1) can be rewritten as

$$\min_{\boldsymbol{x}\in\mathbb{R}^n,\boldsymbol{h}\in\mathbb{R}^m} \Psi(\boldsymbol{x},\boldsymbol{h}) = F(\boldsymbol{x},\boldsymbol{h}) + \mathcal{I}_X(\boldsymbol{x}) + \mathcal{I}_H(\boldsymbol{h}). \tag{2}$$

Problem (2) can be addressed by means of recently proposed optimization methods, provided that the function F and its gradient satisfy some properties that we recall in the following definition.

Definition 1. We define the set $\mathcal{F}$ of functions $F : \mathbb{R}^n \times \mathbb{R}^m \longrightarrow \mathbb{R}$ such that:

(i) $\inf_{\boldsymbol{x}\in\mathbb{R}^n,\boldsymbol{h}\in\mathbb{R}^m} F(\boldsymbol{x},\boldsymbol{h}) > -\infty$;

(ii) for any $\boldsymbol{x}\in\mathbb{R}^n$ and $\boldsymbol{h}\in\mathbb{R}^m$, the partial gradients $\nabla_{\boldsymbol{x}}F(\cdot,\boldsymbol{h})$ and $\nabla_{\boldsymbol{h}}F(\boldsymbol{x},\cdot)$ are globally Lipschitz continuous, i.e., there exist $L_{\boldsymbol{x}}(\boldsymbol{h}),\ L_{\boldsymbol{h}}(\boldsymbol{x}) > 0$ such that

$$\|\nabla_{\boldsymbol{x}}F(\boldsymbol{x}_1,\boldsymbol{h}) - \nabla_{\boldsymbol{x}}F(\boldsymbol{x}_2,\boldsymbol{h})\| \leqslant L_{\boldsymbol{x}}(\boldsymbol{h})\|\boldsymbol{x}_1-\boldsymbol{x}_2\| \quad \forall\boldsymbol{x}_1,\quad \boldsymbol{x}_2\in\mathbb{R}^n,$$
$$\|\nabla_{\boldsymbol{h}}F(\boldsymbol{x},\boldsymbol{h}_1) - \nabla_{\boldsymbol{h}}F(\boldsymbol{x},\boldsymbol{h}_2)\| \leqslant L_{\boldsymbol{h}}(\boldsymbol{x})\|\boldsymbol{h}_1-\boldsymbol{h}_2\| \quad \forall\boldsymbol{h}_1,\quad \boldsymbol{h}_2\in\mathbb{R}^m.$$

Moreover, for each bounded subset $B_1\times B_2$ of $\mathbb{R}^n\times\mathbb{R}^m$, there exist $\alpha(B_1),\ \alpha(B_2) > 0$ such that

$$\sup\{L_{\boldsymbol{h}}(\boldsymbol{x}) : \boldsymbol{x}\in B_1\} \leqslant \alpha(B_1),$$
$$\sup\{L_{\boldsymbol{x}}(\boldsymbol{h}) : \boldsymbol{h}\in B_2\} \leqslant \alpha(B_2);$$