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Dynamics of an impulsive predator–prey logistic population model with state-dependent

Qizhen Xiao, Binxiang Dai *

School of Mathematics and Statistics, Central South University, Changsha, Hunan 410083, PR China

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ABSTRACT

In this paper, the dynamics for a class of state-dependent impulsive predator–prey models with the logistic growth for the predator and prey species are analyzed. By a direct calculation, the existence of a semi-trivial periodic solution is obtained. Based on the geometrical analysis and biological background, the strict threshold value conditions for the existence of positive periodic solutions are depicted. The stabilities of the semi-trivial periodic solution and positive order-1 periodic solutions are proved due to the analogue of Poincaré criterion. Numerical results are carried out to illustrate the feasibility of our main results.

1. Introduction

Over the past few decades, pest control has received growing attention. It is clearly recognized that the pest activities yield major economical stresses on the farmers due to the reduced income. For example, in many countries, the plague of cabbage caterpillar makes a direct fall in crop production. In China, there exists the same phenomenon. The government has to spend much money on controlling the cabbage caterpillar every year.

The issue of the pest management is closely associated with population dynamics, which has been extensively improved by introducing impulsive differential equations (IDEs) [1,2]. In biological population models, many evolution processes are characterized by the fact that at certain moments of time some species experience abrupt changes of states. It is natural to assume that these perturbations act instantaneously, that is, in the form of impulses. Take the pest-epidemic model for example. The density of the susceptible or infectious pest will suddenly change to another level when a biological control strategy, e.g. releasing the infected pest or spraying the microbial pesticide, is taken.

Recently, there has been a significant development in theories of IDEs, especially in the area where impulses are at fixed moments of time. Fox example, Song and Xiang [3], Yang et al. [4], Wang et al. [5] and Liu and Zhang [6] established the conditions for extinction, persistence and the existence and stability of periodic solutions for population dynamical models with the constant impulses. Hui and Chen [7], Tang et al. [8], Wang et al. [9] and Guo and Song [10] studied the population models with the proportional impulses. Georgescu and Morosanu [11], Wang et al. [12] and Baek [13] contained these two impulses for dynamic analyses.

As is well known, impulsive state feedback control strategies are used widely in real world problems. For example, in pest management, control measures will be taken when the pest population reaches the Economic Threshold (the pest population density at which control measures should be undertaken to prevent an increasing pest population from reaching the

* Corresponding author.
E-mail address: bxdai@csu.edu.cn (B. Dai).

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economic injury level which is the lowest pest population density that will cause economic damage). Compared with the impulsive strategy at fixed moments of time, the costs can be reduced and the sustainable control of pests can be expected under the impulsive state feedback control. Therefore, the state-dependent impulsive control strategy is economic, highly efficient and practical.

In recent years, many papers have been devoted to the analysis of mathematical models describing IDEs with state-dependent impulsive effects. For example, Jiang et al. [14] obtained sufficient conditions of the existence, uniqueness and orbitally asymptotic stability of periodic solutions for a class of state-dependent impulsive pest management systems by using the analytical solution. Tang [15,16], Zeng et al. [17], Jiang and Lu [18], Nie et al. [19] considered the dynamic behaviors of predator–prey models with state-dependent impulsive effects and obtained the existence and stability of positive periodic solutions by using the Poincaré map, the Lambert W function and the first integration. Due to the Poincaré map and the method of qualitative analysis, Nie et al. [20] discussed the conditions for existence and stability of positive periodic solutions and showed that there were no periodic solutions with order larger than or equal to three under some conditions for a class of Lotka–Volterra predator–prey models. For more related works, one may refer to the work [21–26] and the references therein. It is notable that all these predator–prey models did not consider the logistic growth for both the predator and prey species which is common in population dynamics. For example, owls, which mainly depend on voles, are also the predators on some small species, such as insects, birds, lizards and fishes. This implies that owls can reproduce in the absence of voles. Motivated by this fact, in this paper, we develop a class of Lotka–Volterra predator–prey models in which both species display the logistic growth.

Let $x(t)$ and $y(t)$ denote the population densities of the prey (pest) and predator (natural enemy) species at time t, respectively. Suppose both populations grow logistically with carrying capacities given by $\frac{r_1}{a_{11}}$ and $\frac{r_2}{a_{22}}$, intrinsic growth rates governed by r_1 and r_2, and the predation rate and conversion coefficient governed by a_{12} and $\frac{a_{21}}{a_{12}}$. The system with the logistic growth for both species is modeled by the following equations:

$$\begin{cases} \left.\begin{aligned} x'(t) &= x(r_1 - a_{11}x - a_{12}y), \\ y'(t) &= y(r_2 + a_{21}x - a_{22}y), \end{aligned}\right\} & x \neq h, \\ \left.\begin{aligned} \Delta x(t) &= -px, \\ \Delta y(t) &= qy + \tau, \end{aligned}\right\} & x = h, \\ x(0) = x_0, \quad y(0) = y_0, \end{cases} \tag{1.1}$$

where x_0 and y_0 denote the initial densities of the prey and predator populations, respectively. The initial condition of system (1.1) can be any point in the non-negative plane $\mathbb{R}_+^2 = \{(x, y) \in \mathbb{R}^2 : x \geqslant 0,\ y \geqslant 0\}$. Throughout this paper we assume that the initial density of the prey population is always less than h. Otherwise, the initial values are taken after an impulsive application. In addition, we assume that $h \in (0, \infty),\ p \in (0, 1)$ and $q \in (-1, \infty)$. When the density of the prey reaches the threshold h at time t_h, control measures are taken and the densities of the prey and predator species immediately become $(1-p)h$ and $(1+q)y(t_h) + \tau$, respectively.

This paper is organized as follows. In the next section, some preliminaries are presented. Then in Section 3, the general criterion for the semi-trivial periodic solution of system (1.1) is stated and proved. Followed in Section 4, the sufficient conditions for the existence and stability of positive periodic solutions of system (1.1) are obtained. These theoretical results are supported with numerical examples in Section 5. Finally, in Section 6, some concluding remarks are derived throughout the whole analysis.

2. Preliminaries

Suppose $p = q = \tau = 0$ for system (1.1). Then we get the following system

$$\begin{cases} x'(t) = x(r_1 - a_{11}x - a_{12}y), \\ y'(t) = y(r_2 + a_{21}x - a_{22}y). \end{cases} \tag{2.1}$$

For system (2.1), it follows from Chen [27] that either the boundary equilibrium $(0, \frac{r_2}{a_{22}})$ is asymptotically stable (if $r_1a_{22} - r_2a_{12} < 0$) or the interior steady state (x^*, y^*) is asymptotically stable (if $r_1a_{22} - r_2a_{12} > 0$), where

$$x^* = \frac{r_1a_{22} - r_2a_{12}}{a_{11}a_{22} + a_{12}a_{21}}, \quad y^* = \frac{r_2a_{11} + r_1a_{21}}{a_{11}a_{22} + a_{12}a_{21}}.$$

Throughout this paper, we assume that the condition $r_1a_{22} - r_2a_{12} > 0$ holds. Obviously, due to Lakshmikantham et al. [28] and Bainov and Simeonov [29], the global existence and uniqueness of solutions of system (1.1) are guaranteed by the smoothness properties of the right-sides of system (1.1).

To discuss the dynamics of system (1.1), we firstly define two cross-sections to the vector field (1.1) by

$$\Sigma^p = \{(x, y) : x = (1-p)h, y > 0\},$$

$$\Sigma^h = \{(x, y) : x = h, y > 0\}.$$

In what follows, we construct a Poincaré map in the usual way [30] and give the definition of periodic solutions.