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ZNN for solving online time-varying linear matrix–vector inequality via equality conversion

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ABSTRACT

In this paper, a special recurrent neural network termed Zhang neural network (ZNN) is proposed and investigated for solving online time-varying linear matrix–vector inequality (LMVI) via equality conversion. That is, by introducing a time-varying vector (of which each element is great than or equal to zero), such a time-varying linear inequality can be converted to a time-varying matrix–vector equation. Then, the ZNN model is developed and investigated for solving online the time-varying matrix–vector equation (as well as the time-varying LMVI) by employing the ZNN design formula. The resultant ZNN model exploits the time-derivative information of time-varying coefficients. Computer-simulation results further demonstrate the efficacy and superiority of the proposed ZNN model for solving online the time-varying LMVI (and the converted time-varying matrix–vector equation).

1. Introduction

In recent years, inequalities (e.g., the Lyapunov matrix inequality) have played a more and more important role in numerous fields of science and engineering applications [1–7]. They (especially linear matrix inequality) have now been viewed as a powerful formulation and design technique for a variety of problems. Being a sub-topic in the research of (linear) inequalities, online solution of linear matrix–vector inequality (LMVI) has been considered to be an essential problem encountered in many science and engineering fields [6–13]. For example, in the redundancy-resolution schemes of robot manipulators, the avoidance feature of joint physical limits (e.g., joint-angle limits, joint-velocity limits and joint-acceleration limits) can be incorporated into the scheme formulation as an inequality constraint [9,10]. Besides, an inequality-based criterion/constraint is proposed, introduced and investigated by Zhang et al. for obstacle-avoidance requirement of redundant robot manipulators [6,7]. By solving such linear matrix–vector inequalities in real time t, the purpose of avoidance feature of joint physical limits and/or environmental obstacles can be achieved successfully [6,7,9,10]. In mathematics, the problem of LMVI can be generally formulated as

$$Ax \leqslant b, \tag{1}$$

where constant coefficient matrix $A \in R^{n\times n}$, vector $b \in R^n$, and $x \in R^n$ is the unknown vector to be obtained.

For online solution of the LMVI [e.g., (1)], two classes of well-developed approaches have been proposed and investigated [4,8,13–16]. One of the classes transforms linear inequalities into the optimization problems (e.g., linear program), which are

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then solved by using classical methods. The second class of approaches is based on iterative methods, one of which is derived from the relaxation method. However, when those approaches are exploited to solve a large-scale system which may have thousands of constraints, they may not be effective and lead to slow convergence [8,14,15]. Because of the parallel distributed nature and the suitability for hardware realization, artificial neural networks have been proposed, developed and studied for scientific research and engineering applications [11–18]. Due to the in-depth research on neural networks, the neural-dynamic approach based on recurrent neural networks has now been viewed as a powerful alternative to online solution of the LMVI [11–13,16]. The classic one is the gradient/gradient-based neural network (GNN) [13,16], which is developed and investigated because of its potential parallel-processing nature and convenience of hardware implementation. Note that such a GNN is usually constructed by defining a scalar-valued lower-bounded energy function and then by exploiting the well-known gradient-descent method [13,14,16].

However, the above-mentioned approaches are designed intrinsically for solving the static (or termed, time-invariant) LMVI. When the coefficients in (1) are time-varying [i.e., $A(t)$ and $b(t)$ with $t \geqslant 0$], these approaches may not work well (instead with large computational errors). In other words, they are less effective and efficient for online time-varying LMVI solving, due to the effects of the time-varying coefficients $A(t)$ and $b(t)$ [14]. Aiming at online time-varying LMVI solving, a special class of recurrent neural networks termed Zhang neural network (ZNN) has been proposed by Zhang et al. [14,15]. By defining vector-valued error functions and employing the ZNN design formula (including its variant), the resultant ZNN models depicted in implicit dynamics exploit methodology time-derivative information of time-varying coefficients, and thus solve the time-varying LMVI efficiently [14,15].

It is worth pointing out that the reported approaches for solving static or time-varying LMVI focus on solving the inequality directly. Since there exist many feasible solutions to the LMVI (no matter whether the coefficients are time-varying or not), it seems complex and difficult to solve directly the (static or time-varying) linear inequality by defining the corresponding scalar-valued energy function or vector-valued error function [13–16]. So, how about converting the (time-varying) LMVI to a (time-varying) matrix–vector equation which can be solved easily and readily (in the sense that lots of methods have been proposed and investigated for solving the matrix–vector equation [19–23])?

In this paper, based on the previous work [14,15], a ZNN model is proposed, developed and investigated for solving online time-varying linear matrix–vector inequality via equality conversion. That is, by introducing a time-varying vector, of which each element is greater than or equal to zero, the time-varying linear inequality is converted to a time-varying equation. Then, by defining the vector-valued indefinite error function and by employing the so-called ZNN design formula [17–21], the ZNN model is developed for solving online the converted time-varying matrix–vector equation, as well as the time-varying LMVI. Note that such a ZNN model depicted in explicit dynamics also exploits the time-derivative information of time-varying coefficients.

The rest of this paper is organized into four sections. In Section 2, the problem formulation of time-varying LMVI is presented, and then such a LMVI is converted to a matrix–vector equation by introducing a time-varying vector. Section 3 develops and investigates the ZNN model for solving online the converted time-varying matrix–vector equation as well as the original time-varying LMVI. In Section 4, computer simulation results of different examples are illustrated, which are synthesized by the proposed ZNN model. Section 5 concludes this paper with final remarks. Before ending this section, it is worth mentioning the main contributions of this paper as follows.

(1) In this paper, the time-varying linear (matrix–vector) inequality is converted to a time-varying (matrix–vector) equation by introducing a time-varying vector, of which each element is greater than or equal to zero. To the best of the authors' knowledge, such an equality conversion has not yet been investigated by other researchers.
(2) This paper proposes and develops the ZNN model for solving online the time-varying LMVI (and its converted matrix–vector equation) by defining the vector-valued indefinite error function and then by employing the ZNN design formula.
(3) Computer simulation results of different examples are illustrated to demonstrate the efficacy and superiority of the proposed ZNN model for online time-varying LMVI (and its converted equation) solving.

2. Problem formulation and conversion

In this section, to lay a basis for further discussion, the problem formulation of time-varying LMVI is presented. Then, the conversion from LMVI to matrix–vector equation is discussed by introducing a time-varying vector.

2.1. Problem formulation

In this paper, we consider the following problem of time-varying LMVI [14,15]:

$$A(t)x(t) \leqslant b(t), \tag{2}$$

where $A(t) \in R^{n\times n}$ and $b(t) \in R^n$ are, respectively, smoothly time-varying matrix and vector, and $x(t) \in R^n$ is the unknown vector to be obtained. The objective of this paper is to find a feasible solution $x(t)$ such that (2) holds true for any time instant $t \geqslant 0$. Note that, for further discussion, $A(t)$ is assumed to be nonsingular at any time instant $t \in [0, +\infty)$ in this paper.