Applied Mathematics and Computation 259 (2015) 480–489

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Mild solutions for class of neutral fractional functional differential equations with not instantaneous impulses

Ganga Ram Gautam *, Jaydev Dabas

Department of Applied Science and Engineering, IIT Roorkee, Saharanpur Campus, Saharanpur 247001, India

ARTICLE INFO

Keywords:
Fractional order differential equation
Fixed point theorems
Initial conditions
Impulse

ABSTRACT

This article contains the existence results of the mild solutions of an abstract semilinear neutral fractional differential equations with not instantaneous impulses. The results are proved by using the theory of analytic α-resolvent family and fixed point theorems. One application involving partial differential equations with impulses are presented.

1. Introduction

There are several fields of research in which the topic of fractional differential equations have recently emerged as an important tools to model real world problems. It has great applications in several disciplines and various fields of science such as physics, polymer rheology, regular variation in thermodynamics, biophysics, blood flow phenomena, aerodynamics, electrodynamics of complex medium, visco-elasticity, electrical circuits, electron-analytical chemistry, biology, control theory and fitting of experimental data, etc. For more details one can see [17,18] and references therein.

Neutral type differential equations mainly look as models of electrical network arise in high speed computers, for example these are used to interconnect switching circuits. For further detail of such type neutral differential equations we refer the reader to see the papers [3,7,12,15,16] and reference therein.

Fractional order functional differential equations originate in several fields of applied mathematics, science and engineering problems. Recently fractional functional differential equations with state dependent delay seems frequently in many fields as modeling of equations, panorama of natural phenomena and porous media. See for more details in the cited papers [1–7,13–15].

The differential equations with impulsive effects have been appeared as in natural description evolution processes. The impulsive effect can be shown in many biological phenomena involving thresholds, bursting rhythm models in medicine and biology, optimal control model in economics, pharmacokinetics and frequency modulated system etc. See the cited papers [1,3,8–12,19–21] for more detail of this topic.

The differential equation with not instantaneous impulsive condition first time used by the author's [20] for the following abstract problem

$$u'(t) = Au(t) + f(t, u(t)), \quad t \in (s_i, t_{i+1}], \quad i = 0, 1, \dots, N, \tag{1}$$
$$u(t) = g_i(t, u(t)), \quad t \in (t_i, s_i], \quad i = 1, 2, \dots, N, \tag{2}$$
$$u(0) = x_0, \tag{3}$$

* Corresponding author.
E-mail addresses: gangaiitr11@gmail.com (G.R. Gautam), jay.dabas@gmail.com (J. Dabas).

http://dx.doi.org/10.1016/j.amc.2015.02.069
0096-3003/

where $A : D(A) \subset X \to X$ is the generator of a C_0-semigroup of bounded operators $\{T(t)\}_{t\geqslant 0}$ defined on a Banach space X. In [20], authors have introduced the concepts of mild and classical solution and established the existence results for class of these types of the problems by using fixed point theorems.

Further, Pierri et al. [21] extend the results of [20] in the study of the problem (1)–(3) using the theory of analytic semigroup and fractional power of closed operators and established the existence results of solutions.

Motivated by the above said work of [20,21], we consider the following fractional functional differential equations with not instantaneous impulses:

$$^{C}D_t^{\alpha}\, Q\left(y_{\rho(t,y_t)}\right) = AQ\left(y_{\rho(t,y_t)}\right) + f\left(t, y_{\rho(t,y_t)}, By_{\rho(t,y_t)}\right), \quad t \in (s_i, t_{i+1}], \quad i = 0, 1, \ldots, N, \tag{4}$$

$$y(t) = g_i(t, y(t)), \quad t \in (t_i, s_i], \quad i = 1, 2, \ldots, N, \tag{5}$$

$$y(t) = \phi(t), \quad t \in (-\infty, 0], \tag{6}$$

where $^{C}D_t^{\alpha}$ is Caputo's fractional derivative of order $0 < \alpha \le 1$ and operational interval $J = [0, T]$. The map $A : D(A) \subset X \to X$ is the a closed linear sectorial operator defined on a Banach space $(X, \|\cdot\|)$, $0 = t_0 = s_0 < t_1 \leqslant s_1 \leqslant t_2 < \ldots < t_N \leqslant s_N \leqslant t_{N+1} = T$, are pre-fixed numbers, $g_i \in C((t_i, s_i] \times X; X)$ for all $i = 1, 2, \ldots, N$. $f : J \times \mathfrak{B}_h \times \mathfrak{B}_h \to X$, $Q(y_{\rho(t,y_t)}) = y(t) + h(t, y_{\rho(t,y_t)})$, $Q(\phi) = \phi(0) + h(0, \phi)$ and $\rho : J \times \mathfrak{B}_h \to (-\infty, T]$ are appropriate functions. The history function $y_t : (-\infty, 0] \to X$ is element of $\mathfrak{B}_h$ and defined by $y_t(\theta) = y(t+\theta)$, $\theta \in (-\infty, 0]$ and the function ϕ also belong to $\mathfrak{B}_h$. The term $B(y_{\rho(t,y_t)})$ is given by $B(y_{\rho(t,y_t)}) = \int_0^t K(t,s)(y_{\rho(s,y_s)})ds$, where $K \in C(D, \mathbb{R}^+)$, is the set of all positive functions which are continuous on $D = \{(t,s) \in \mathbb{R}^2 : 0 \leqslant s \leqslant t < T\}$ and $B^* = \sup_{t\in[0,t]} \int_0^t K(t,s)ds < \infty$. Here impulses are not instantaneous means these impulses start abruptly at the points t_i and their action continues on the interval $[t_i, s_i]$ and $\mathfrak{B}_h$ stand for phase space defined in next section.

This paper is divided into four sections, in which second section provides some basic definitions, notations and propositions. In third section, we obtain the existence results of the mild solutions of the considered problem (4)–(6). The fourth section is concerned with an example.

2. Preliminaries

Let $(X, \|\cdot\|_X)$ be a complex Banach space of functions with the norm $\|y\|_X = \sup_{t\in J}\{|y(t)| : y \in X\}$ and $L(X)$ denotes the Banach space of bounded linear operators from X into X equipped with its natural topology. Due to infinite delay we use abstract phase space $\mathfrak{B}_h$ as defined in [11] details are as follow:

Assume that $h : (-\infty, 0] \to (0, \infty)$ is a continuous functions with $l = \int_{-\infty}^0 h(s)ds < \infty, t \in (-\infty, 0]$. For any $a > 0$, we define

$$\mathfrak{B} = \{\psi : [-a, 0] \to X \text{ such that } \psi(t) \text{ is bounded and measurable}\}$$

and equipped the space $\mathfrak{B}$ with the norm $\|\psi\|_{[-a,0]} = \sup_{s\in[-a,0]} \|\psi(s)\|_X$, $\forall\, \psi \in \mathfrak{B}$. Let us define

$$\mathfrak{B}_h = \left\{\psi : (-\infty, 0] \to X, \text{ s.t. for any } a \geqslant c > 0, \psi|_{[-c,0]} \in \mathfrak{B} \;\&\; \int_{-\infty}^0 h(s)\|\psi\|_{[s,0]}ds < \infty\right\}.$$

If $\mathfrak{B}_h$ is endowed with the norm $\|\psi\|_{\mathfrak{B}_h} = \int_{-\infty}^0 h(s)\|\psi\|_{[s,0]}ds$, $\forall\, \psi \in \mathfrak{B}_h$, then it is clear that $(\mathfrak{B}_h, \|\cdot\|_{\mathfrak{B}_h})$ is a complete Banach space.

To treat the impulsive conditions, we consider the space

$$\mathfrak{B}'_h := PC((-\infty, T]; X), \quad T < \infty,$$

be a Banach space of all such functions $y : (-\infty, T] \to X$, which are continuous every where except for a finite number of points $t_i \in (0, T)$, $i = 1, 2, \ldots, N$, at which $y(t_i^+)$ and $y(t_i^-)$ exists and endowed with the norm

$$\|y\|_{\mathfrak{B}'_h} = \sup\left\{\|y(s)\|_X : s \in [0, T]\right\} + \|\phi\|_{\mathfrak{B}_h}, \quad y \in \mathfrak{B}'_h,$$

where $\|\cdot\|_{B'_h}$ to be a semi-norm in $\mathfrak{B}'_h$.

For a function $y \in \mathfrak{B}'_h$ and $i \in \{0, 1, \ldots, N\}$, we introduce the function $\bar{y}_i \in C([t_i, t_{i+1}]; X)$ given by

$$\bar{y}_i(t) = \begin{cases} y(t), \text{ for } t \in (t_i, t_{i+1}], \\ y(t_i^+), \text{ for } t = t_i. \end{cases}$$

If $y : (-\infty, T] \to X$ such that $y \in \mathfrak{B}'_h$ is continuous then for all $t \in J$, the following conditions hold:

(C_1) $y_t \in \mathfrak{B}_h$.
(C_2) $\|y(t)\|_X \leqslant H\|y_t\|_{\mathfrak{B}_h}$.