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Delay-dependent synchronization for non-diffusively coupled time-varying complex dynamical networks

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ABSTRACT

This paper investigates the delay-dependent synchronization schemes for the non-diffusively coupled time-varying complex dynamical networks. The outer coupling configuration matrix in our network model may be non-diffusive, time-varying, uncertain, asymmetric and irreducible. Different time-varying coupling delays for different nodes are also put into consideration in this paper. Besides, the nodes may have different state dimensions. Furthermore, only the common bound of the outer coupling coefficients (CBOCC) is used to design the synchronization controllers. If the CBOCC is known, our delay-dependent synchronization scheme can guarantee the network achieving exponential synchronization. And when the CBOCC is uncertain, the adaptive synchronization scheme, where only one adaptive law is needed, is proposed to guarantee the network realizing asymptotic synchronization. Simulation examples are provided to verify the effectiveness and feasibility of our theoretical results.

1. Introduction

Complex dynamical networks have attracted significant attention from various fields for their extensive uses in modeling many natural and artificial systems. Synchronization, as one of the typical collective behaviors and basic motions of complex dynamical networks, has been widely investigated during the past few decades for both time-invariant and time-varying networks [1–27]. Many synchronous phenomena such as the synchronous transfer of digital or analog signals in communication networks are very important and useful in our daily life, so it is essential to find conditions that guarantee the nodes in a network converging to the same desired trajectory asymptotically or exponentially, that is, the network achieving asymptotic or exponential synchronization. To synchronize a complex dynamical network, an important method is adding proper controllers to its nodes. Hitherto, many control methods, such as integral control [4] and decentralized dynamical compensation control [12,13], have been proposed to synchronize the networks.

However, a basic assumption adopted by all the aforementioned contributions is that the nodes of the network are diffusively coupled, which is not always in agreement with the natural world. Rather than being simply diffusive, the coupling functions between nodes in the network are typically nonlinear with a functional form that has been engineered by nature to best fulfill a certain need [9]. To the best of our knowledge, few literatures have focused on the synchronization of the non-diffusively coupled complex dynamical networks. Thus, to be more consistent with realist networks, it is important to investigate the synchronization schemes for the complex dynamical networks with non-diffusively coupled nodes in this paper.

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Another assumption adopted by most of the existing contributions about the synchronization of complex dynamical networks is that the networks possess static and exact outer coupling structures, which is also not always coincident with the real-world networks. In fact, time-varying or uncertain outer coupling relationships exist in a lot of realist networks, especially in biological and engineering ones. Hence, it is also essential to study the synchronization for the networks with time-varying or uncertain coupling structures. For such networks, some synchronization schemes have been designed [15–21]. For example, a time-varying complex dynamical network model was introduced and its synchronization phenomenon was also investigated in [18]. For a class of uncertain complex dynamical networks, several adaptive synchronization criteria have been deduced in [15]. Global bounded synchronization problem of general dynamical networks with nonidentical nodes and time-varying outer coupling symmetric configuration was addressed in [17]. But it should be noted that all the synchronization criteria proposed in [15–21] are only valid for the networks with diffusively coupled nodes. Therefore, it is necessary for this paper to further investigate the synchronization problem for the networks with non-diffusively coupled nodes and unknown or time-varying couplings.

It is worth pointing out that the corresponding nodes of many real-world networks possess almost the same qualities of nature or appearance or function [10–13]. We call these qualities as similarities and the nodes are named as similar nodes in this paper. In general, each node in the network has some other different characters except for the common qualities. Therefore, similar nodes may be nonidentical and it is important and reasonable to describe the similarities of the nodes based on their dynamics no matter their state dimensions are different or not. The nodes with the same dynamics are the special case of similar nodes. A complex dynamical network with different dimensional similar nodes will exhibit much more complicated behaviors than those with the nodes of the same state dimension. The investigation for the synchronization of the networks with different dimensional similar nodes is still in an initial stage. For such networks, although Refs. [10–13] have proposed some synchronization schemes, they are only valid for those with diffusively coupled nodes and static topological structures. Hence, it is necessary to explore new synchronization schemes for the time-varying or uncertain networks with non-diffusively coupled similar nodes in this paper.

As is well known, time delays are ubiquitous in nature and engineering. Ignoring them may lead to incorrect synchronization criteria. And generally, time delays of different nodes are generally nonidentical and time-varying. Although [14,22–25] have proposed several synchronization criteria for delayed networks, they will be invalid for the non-diffusively coupled time-varying networks with different dimensional similar nodes and different time-varying coupling delays.

Inspired by the aforementioned discussions, it is necessary to further investigate the delay-dependent synchronization schemes for the time-varying complex dynamical networks with non-diffusively coupled nodes and different time-varying coupling delays in this paper. Compared with the delay-independent controllers, delay-dependent control schemes are less conservative since they contain information on the length of delays [28].

The rest of this paper is organized as follows. Section 2 introduces a non-diffusively coupled time-varying complex dynamical network model with different time-varying coupling delays and similar nodes. Some assumptions are also made in this section. Then, in Section 3, based on the Lyapunov–Krasovskii stability theory and Barbalat's lemma, decentralized delay-dependent synchronization controllers are synthesized for the network with known or unknown CBOCC. Section 4 provides some illustrative examples to verify the effectiveness and advantage of our theoretical results in this paper. Finally, conclusions are given in Section 5.

2. Network model and preliminaries

Consider a general non-diffusively coupled time-varying complex dynamical network composed of N different delayed coupling nodes. The entire network is described as:

$$\dot{x}_i = f_i(x_i) + \sum_{\substack{j=1 \\ j\neq i}}^{N} c_{ij}(t)\big(h_{ij}(x_j(t-\tau_j(t))) - h_{ii}(x_i(t-\tau_i(t)))\big) + G_i(x_i)u_i \tag{1}$$

where $i = 1,2,\ldots,N$, $x_i = (x_{i1}, x_{i2}, \ldots, x_{in_i})^T \in R^{n_i}$ is the state vector of node i, n_i is its state dimension. For simplicity, let $n_1 = \min_{1\le k\le N}\{n_k\}$; $f_i(x_i) \in R^{n_i}$, $u_i \in R^{n_1}$ and $G_i(x_i) \in R^{n_i\times n_1}$ are the smooth nonlinear vector field, the control input and the control gain function matrix of node i, respectively; $h_{ij}(x_j(t-\tau_j(t)))$: $R^{n_j} \to R^{n_i}$ $(i, j = 1,2,\ldots,N)$ are delayed inner coupling function vectors; $\tau_i(t)$ is the time-varying coupling delay of node i; $C(t) = (c_{ij}(t))_{N\times N}$ is the time-varying outer coupling configuration matrix representing the coupling strength and the topological structure of the network at time t, where $c_{ij}(t)$ is defined as follows: If there is an interconnection from node i to node j $(j \neq i)$ at time t, then $c_{ij}(t) \neq 0$, otherwise, $c_{ij}(t) = 0$.

Remark 1. It is worth pointing out that the nodes in network (1) may be non-diffusively coupled, which is remarkably different from almost all the existing contributions about network synchronization. Besides, the time-varying coupling delays $\tau_i(t)$ $(i = 1,2,\ldots,N)$ may be different, which is more coincident with the realist networks. Especially, if the outer coupling matrix $C(t)$ and delays $\tau_i(t)$ in network (1) are diffusively coupled constant matrix and constant delays, then the network (1) becomes that in Ref. [13].

Throughout this paper, we make the following assumptions on network (1).