Applied Mathematics and Computation 259 (2015) 540–551

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A general decay result of a nonlinear system of wave equations with infinite memories

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ARTICLE INFO

Keywords:
General decay
Infinite memory
Nonlinear
Wave equation
Viscoelasticity

ABSTRACT

In this paper, we consider a system of two wave equations with nonlinear damping and source terms acting in both equations in the presence of infinite-memory terms and prove an explicit and general decay result. Our approach allows a wide class of kernels, among which those of exponential decay type are only special cases.

1. Introduction

In this paper, we consider the following system

$$\begin{cases} u_{tt}(x,t) - \Delta u(x,t) + \int_0^{+\infty} g(s)\Delta u(x,t-s)ds + \lambda \mid u_t|^{m-1}u_t = f_1(u,v) & \text{in } \Omega\times(0,\infty), \\ v_{tt}(x,t) - \Delta v(x,t) + \int_0^{+\infty} h(s)\Delta v(x,t-s)ds + \mu \mid v_t|^{r-1}v_t = f_2(u,v) & \text{in } \Omega\times(0,\infty), \\ u(x,t) = v(x,t) = 0, & \text{in } \partial\Omega\times(0,\infty), \\ u(x,-t) = u_0(x,t),\ u_t(x,0) = u_1(x),\ v(x,-t) = v_0(x,t),\ v_t(x,0) = v_1(x), & \text{in } \Omega\times(0,\infty), \end{cases} \tag{1.1}$$

$$\begin{cases} f_1(u,v) = a \mid u+v|^{2(\rho+1)}(u+v) + b \mid u|^{\rho}u \mid v|^{\rho+2}, \\ f_2(u,v) = a \mid u+v|^{2(\rho+1)}(u+v) + b \mid v|^{\rho}v \mid u|^{\rho+2}, \end{cases} \tag{1.2}$$

where u and v denote the transverse displacements of waves, Ω is a bounded domain of $\mathbb{R}^N (N \geq 1)$ with a smooth boundary $\partial\Omega,\ \rho,\ m,\ r,\ \lambda,\ \mu$ are positive constants, the kernels g and h are satisfying some conditions to be specified later and the nonlinear coupling functions describe the interaction between the two waves and can be considered as a slight modification of the nonlinearity appeared in the well-known Klein–Gordon system [1,2,18,19]

$$\begin{cases} u_{tt} - \Delta u + m_1 u + k_1 u v^2 = 0, \\ v_{tt} - \Delta v + m_2 v + k_2 u^2 v = 0. \end{cases}$$

During the last half century, this type of problems has attracted a lot of attention and many results of existence, stability and blow up have been established. We start with the pioneer work of Dafermos [7,8] where he considered certain one-dimensional viscoelastic problems of the form

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http://dx.doi.org/10.1016/j.amc.2015.02.085
0096-3003/

$$\begin{cases} \rho u_{tt}(x,t) = cu_{xx}(x,t) - \int_{-\infty}^{t} g(t-s)u_{xx}(x,s)ds, & x \in [0,1], t \in [0,\infty), \\ u(0,t) = u(1,t) = 0, & t \in (-\infty,\infty). \end{cases}$$

He established various existence results and then proved, for smooth monotone decreasing relaxation functions g, that the solutions go to zero as t goes to infinity. However, no rate of decay has been specified. In [5], Cavalcanti et al. considered

$$u_{tt} - \Delta u + \int_0^t g(t-s)\Delta u(x,s)ds + a(x)u_t + |u|^{p-1}u = 0, \quad \text{in } \Omega \times (0,\infty),$$

where $a : \Omega \to \mathbb{R}^+$ is a function which may vanish on a part of the domain Ω but satisfies $a(x) \geqslant a_0$ on $\omega \subset \Omega$ and g satisfies, for two positive constants ξ_1 and ξ_2,

$$-\xi_1 g(t) \leqslant g'(t) \leqslant -\xi_2 g(t), \quad t \geqslant 0$$

and established an exponential decay result under some restrictions on ω. Berrimi and Messaoudi [4] discussed

$$u_{tt} - \Delta u + \int_0^t g(t-s)\Delta u(s)ds = |u|^{\gamma}u, \quad x \in \Omega, \quad t \geqslant 0$$

and established the result of [5], under weaker conditions on the relaxation function. Fabrizio and Polidoro [9] studied the following system

$$\begin{cases} u_{tt} - \Delta u + \int_0^t g(t-\tau)\Delta u(\tau)d\tau + u_t = 0, & \text{in } \Omega \times (0,\infty), \\ u = 0, & \text{on } \partial\Omega \times (0,\infty) \end{cases}$$

and showed that the exponential decay of the relaxation function is a necessary condition for the exponential decay of the solution energy.

For viscoelastic systems, Andrade and Mognon [2] treated the following problem

$$\begin{cases} u_{tt} - \Delta u + \int_0^t g(t-s)\Delta u(s)ds + f_1(u,v) = 0, & \text{in } [0,T] \times \Omega, \\ v_{tt} - \Delta v + \int_0^t h(t-s)\Delta v(s)ds + f_2(u,v) = 0, & \text{in } [0,T] \times \Omega \end{cases} \tag{1.3}$$

with

$$f_1(u,v) = |u|^{p-2}u\,|v|^p \quad \text{and} \quad f_2(u,v) = |v|^{p-2}v\,|u|^p,$$

where $p > 1$ if $n = 1,2$ and $1 < p \leqslant \frac{n-1}{n-2}$ if $n \geqslant 3$. They proved the well posedness for the problem under the following assumptions on the relaxation functions:

$$\begin{cases} 1 - \int_0^{+\infty} g(s)ds > 0, \quad 1 - \int_0^{+\infty} h(s)ds > 0, \\ g'', h'' \in L^1(0,\infty) \end{cases}$$

and for some positive constants α and β

$$-\alpha g(t) \leqslant g'(t) \leqslant -\beta g(t)$$

and

$$-\alpha h(t) \leqslant h'(t) \leqslant -\beta h(t).$$

In [21], Santos considered (1.3) with

$$f_1(u,v) = a(u-v) \quad \text{and} \quad f_2(u,v) = -a(u-v),$$

where a is a positive constant and the relaxation functions satisfy

$$\begin{cases} -a_1 g^p(t) \leqslant g'(t) \leqslant -a_2 g^p(t), \\ 0 \leqslant g''(t) \leqslant \gamma g^p(t) \end{cases}$$

and

$$\begin{cases} -a_1 h^p(t) \leqslant h'(t) \leqslant -a_2 h^p(t), \\ 0 \leqslant h''(t) \leqslant \gamma h^p(t) \end{cases}$$

for some $1 \leqslant p < 2$. He proved an exponential decay result for the kernels decaying exponentially, and a polynomial decay result for the kernels decaying polynomially.