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Spatial reciprocity for discrete, continuous and mixed strategy setups

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ABSTRACT

The existence of cooperation in the social dilemma has been extensively studied based on spatial structure populations, namely, the so-called spatial reciprocity. However, vast majority of existing works just simply presume that agents can offer the discrete choice: either the cooperative (C) or defective (D) strategy, which, to some extent, seems unrealistic in the empirical observations since actual options might be continuous, mixed rather than discrete. Here, we propose discrete, continuous and mixed strategy setups in the social dilemma games and further explore their performance on network populations. Interestingly, it is unveiled that there is actually considerable inconsistency in terms of equilibrium among different strategy games. Furthermore, we reveal how different cooperative arrangements among these three strategy setups can be established, depending on whether the presumed dilemma subclass is a boundary game between prisoner's dilemma game and Chicken game or between prisoner's dilemma game and Stag-Hunt game.

1. Introduction

Understanding the emergence and maintenance of collective cooperation among the ensembles of selfish individuals represents one of the most fascinating challenges in both evolutionary biology and social science. Aiming to elucidate this long-standing issue, evolutionary game theory, which provides a useful theoretical framework, has been extensively investigated from different disciplines [1–3]. As the typical metaphors, the pair-wise interaction games, like prisoner's dilemma (PD) game and snowdrift (SD) game (or Chicken game (CH)) [4,5], have attracted great attention by borrowing the quantitative analysis technology [6]. However, in the well-mixed population, these paradigmatic models do not support the organization of cooperation dynamics. To overcome this unfavorable outcome, a great number of scenarios need to be identified that can offset the unbeneficial situations of social dilemma and promote the evolution of cooperation [7–9]. In particular, Nowak recently attributed these achievements to five mechanisms: kin selection, direct reciprocity, indirect reciprocity, network reciprocity and group selection [10], these mechanisms can be somewhat related to the reduction of an opposing player's anonymity relative to the existing well-mixed situation.

Among the foregoing five mechanisms, network reciprocity, where players are arranged on the spatially structured topology and interact only with their direct neighbors, has attracted the most notable interest (see Refs. [7,9,11] for recent

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review), since cooperators can survive by means of forming compact clusters, which minimize the exploitation by defectors and protect those cooperators that are located in the interior of such clusters [12]. After this seminal finding, the role of spatial structure and its various underlying promoting mechanisms, in evolutionary games, have been substantially explored. Most notably, they includes complex networks [13–26], social diversity [25,27,28], punishment and reward mechanism [29,30] diluted environment [31], simultaneous adoption of different strategies depending on the opponents [32], the mobility of players [33–38], heterogeneous activity or ability [39–41], differences in evolutionary time scales [42,43], voluntary participation [44–46] and coevolutionary selection of dynamical rules [9,47–49], to name but a few. We can also look at one recent example more specifically. In [50], where two interdependent networks were correlated via a utility function, the authors found that an intermediate density of interactions between networks warranted an optimal cooperation condition, beyond the content of traditional spatial reciprocity [51] (for more details about recent development of evolutionary games on multilayer networks, we can also refer to [52]).

In spite of numerous achievements of the recent years, vast majority of existing works simply assume that agents only take the strategy: pure cooperation (C) or pure defection (D), which seems inconsistent with the empirical cases in the realistic life. In the society, people are more likely to regard their decision as a continuous or mixed (both are determined with the certain probability) rather than discrete decision [53–64]. Recent works have shown that when the payoff of the continuous strategy is a linear function, this kind of setup leads to the identical equilibrium with discrete protocol in the infinite well-mixed population [59–61]. In addition, another investigation reports that a finite size of population can bring obviously different equilibria for continuous and discrete setups [65]. However, these finding are mainly based on the finite well-mixed population, which do not involve any spatial interaction. An interesting and crucial question thus poses itself, which we aim to address in what follows. If complex network is considered as the underlying interaction topology, do the setups of continuous and mixed strategies change the initial equilibrium of discrete strategy? Furthermore, do they promote or hinder the evolution of cooperation?

This study uses comprehensive, systematic numerical simulations to examine whether inconsistency among discrete, continuous, and mixed strategies exists in a spatially structured population.

2. Evolutionary models

2.1. Discrete, continuous and mixed strategy setups

In this work, we consider the pair-wise interaction game as the archetype. When the discrete strategy is presumed, two players can simultaneously make the decision between cooperation (C) and defection (D). If both cooperate (defect) they receive the reward R (the punishment P). If, however, one chooses cooperation while the other defects, the later gets the temptation T and the cooperator is left the sucker's payoff S. For simplicity, yet without loss of generality, the payoffs can be rescaled as $R = 1$, $D_r = P - S$, $D_g = T - R$ and $P = 0$, and then payoff matrix becomes the following expression [66],

$$G = \begin{pmatrix} R & S \\ T & P \end{pmatrix} = \begin{pmatrix} 1 & -D_r \\ 1 + D_g & 0 \end{pmatrix}, \tag{1}$$

where D_r and D_g are the dilemma strength from the Stag-Hunt (SH) type and the Chicken (CH) type [7], respectively. Interestingly, different values of both D_r and D_g in the parameter plane usually lead to various classes of games. If both D_g and D_r are positive, it falls into prisoner's dilemma (PD) game, where defection completely dominates the system. When both of them are negative, it belongs to a trivial (TR) game having no dilemma, since mutual cooperation is the optimal strategy choice. If D_g is positive while D_r negative, it is called Chicken (CH) game, which has an internal polymorphic equilibrium. Lastly, the opposite case (negative D_g and positive D_r) is classified into Stag-Hunt (SH) game and reverts to a bi-stable equilibrium.

With respect to continuous strategy, we assign each player i a random parameter s_i in the interval $[0, 1]$ to denote its strategy (as well the probability of cooperation). This setting is performed uniformly before the formal interaction. When player i plays the game with agent j, it can obtain the following payoff

$$\pi(s_i, s_j) \equiv (S - P)s_i + (T - P)s_j + (P - S - T + R)s_i s_j + P = -D_r \cdot s_i + (1 + D_g) \cdot s_j + (-D_g + D_r) \cdot s_i \cdot s_j. \tag{2}$$

This might be the simplest and most plausible setup, which expands the discrete strategy scenario but still uses the elementary payoff matrix.

When the mixed strategy is presummed, each agent i is still assigned with a real number $s_i \in [0, 1]$ as what to do in the following step. However, different from continuous proposal, the player can only offer either pure cooperation or defection as its strategy. That is to say, the player chooses pure cooperation with probability s_i, otherwise it takes defection.

2.2. Simulation setting

Through this work we put more attention to the spatial reciprocity in the prisoner's dilemma (PD) game, where $0 \leq D_r \leq 1$ and $0 \leq D_g \leq 1$ in Eqs. (1) and (2) (also see Appendix D), since the organization of cooperation seems most difficult in this type of social dilemma. As the interaction networks, we use seven types of topology structure: (i) cycle; (ii) square lattice; (iii) homogeneous small world network (Ho-SW), which is made from a cycle graph by replacing several links with