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An efficient algorithm for solving nonlinear Volterra–Fredholm integral equations

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ABSTRACT

In this work, we develop a new effective method for solving nonlinear Volterra–Fredholm integral equation. The existence of any ε-approximate solution is proved. At the same time, an effective method for obtaining the ε-approximate solution is established. The final numerical examples illustrate that our approach is valid not only for weakly nonlinear problems but also for strongly nonlinear problems.

1. Introduction

The nonlinear Volterra–Fredholm integral equations arise from various physical and biological models. The essential features of these models are of wide applicable [1]. In this work, we consider the nonlinear Volterra–Fredholm integral equation given by

$$\mathbb{L}u(x) = u(x) + \lambda_1 \int_a^x K_1(x,t)N_1(u(t))dt + \lambda_2 \int_a^b K_2(x,t)N_2(u(t))dt = f(x), \tag{1}$$

where λ_1, λ_2 are real constants, N_1, N_2 are nonlinear functions, K_1, K_2, u are all continuous functions and u is the unknown function.

In the last decade, nonlinear Volterra–Fredholm integral equations receives widespread attentions. And many methods have been proposed for solving them such as modified decomposition method [2,3], reproducing kernel Hilbert space method [4], Legendre wavelets method [5], homotopy perturbation method [6], a composite collocation method [7], rationalized Haar functions method [8], variational iteration method [9], collocation method based on radial basis functions [10], method based on Bernstein operational matrices [11], hybrid of block-pulse functions and Taylor series method [12], sinc method [13] etc. The comprehensive view of nonlinear Volterra–Fredholm integral equations can be found in Ref. [14].

In this work, we propose a new method based on spline function and ε-approximate solution. This method is simple, effective and easy to implement. It's worth noting that numerical experiments show that this method is still valid for strongly nonlinear problems.

The rest of the paper is organized as follows. Section 2 give the main results. In Section 3, the numerical results confirm that the algorithm is accurate, efficient and readily implemented. Section 4 ends this paper with a brief conclusion.

2. The ε-approximate solution of Eq. (1)

Definition 2.1. Let $\varepsilon > 0$. v is called an ε-approximate solution of (1) if $\|\mathbb{L}v - f\|_C \leqslant \varepsilon$.

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In this work, we divide interval $[a,b]$ into $3n$ subintervals $[x_{i-1},x_i]$, where $x_i = a + \frac{(b-a)i}{3n}$, $i = 0,1,2,\ldots,3n$. Suppose that $u(x)$ is sufficiently smooth and define its interpolation function $S_1(x)$ as follows

$$S_1(x) = \begin{cases} S_{11}(x), & x \in [x_0,x_3] \\ S_{12}(x), & x \in [x_3,x_6] \\ \vdots & \\ S_{1k}(x), & x \in [x_{3k-3},x_{3k}] \\ \vdots & \\ S_{1n}(x), & x \in [x_{3n-3},x_{3n}] \end{cases},$$

where

$$S_{1k}(x) = \sum_{j=0}^{3} l_{kj}(x)u(x_{kj}).$$

and $l_{kj}(x) = \prod_{\substack{i=3k-3\\ i\neq 3(k-1)+j}}^{3k} \frac{x-x_i}{x_j-x_i}$, $\quad x_{kj} = x_{3k-3+j}$ and l_{kj} are called Lagrange interpolation basis functions.

Lemma 2.1. *Assume that $u(x) \in C^4[a,b]$ and $S_1(x)$ is the equidistant Lagrange interpolation function of $u(x)$, $h = \frac{b-a}{3n}$, then*

$$\max_{a\leqslant x\leqslant b}|u(x) - S_1(x)| \leqslant \frac{1}{24}\max_{a\leqslant x\leqslant b}|u^{(4)}(x)|h^4. \tag{2}$$

Proof. When $x \in [x_{3k-3},x_{3k}]$, using the error estimation of the equidistant Lagrange interpolation function, we have

$$|u(x) - S_1(x)| = \left|\frac{t(t-1)(t-2)(t-3)}{4!}h^4u^{(4)}(\xi)\right|,$$

where $\xi \in (x_{3k-3},x_{3k}) \subset [a,b]$ and $0 \leqslant t \leqslant 3$. Inequality

$$|t(t-1)(t-2)(t-3)| \leqslant 1, \quad 0 \leqslant t \leqslant 3,$$

gives

$$|u(x) - S_1(x)| \leqslant \frac{1}{24}\max_{a\leqslant x\leqslant b}|u^{(4)}(x)|h^4.$$

So, the conclusion follows. □

Put $M = \max\{\int_a^b |K_1(x,t)|dt, \int_a^b |K_2(x,t)|dt\}$.

Lemma 2.2. *Let $u(x)$ be the exact solution of Eq. (1), $S_1(x), S_2(x), S_3(x)$ are the equidistant Lagrange interpolation functions of $u(x), N_1(u(x)), N_2(u(x))$, respectively. That is,*

$$S_i(x) = \begin{cases} S_{i1}(x), & x \in [x_0,x_3] \\ S_{i2}(x), & x \in [x_3,x_6] \\ \quad\vdots & \\ S_{ik}(x), & x \in [x_{3k-3},x_{3k}] \\ \quad\vdots & \\ S_{in}(x), & x \in [x_{3n-3},x_{3n}] \end{cases}, \quad i = 1,2,3,$$

and

$$S_{1k}(x) = \sum_{j=0}^{3} l_{kj}(x)u(x_{kj}),$$

$$S_{2k}(x) = \sum_{j=0}^{3} l_{kj}(x)N_1(u(x_{kj})),$$

$$S_{3k}(x) = \sum_{j=0}^{3} l_{kj}(x)N_2(u(x_{kj})),$$