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A classification of one-dimensional cellular automata using infinite computations

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ABSTRACT

This paper proposes an application of the Infinite Unit Axiom and *grossone*, introduced by Yaroslav Sergeyev (see Sergeyev (2003, 2009, 2013, 2008, 2008) [15–19]), to classify one-dimensional cellular automata whereby each class corresponds to a different and distinct dynamical behavior. The forward dynamics of a cellular automaton map are studied via defined classes. Using these classes, along with the Infinite Unit Axiom and grossone, the number of configurations that equal those of a given configuration, in some finite central window, under an automaton map can now be computed. Hence a classification scheme for one-dimensional cellular automata is developed, whereby determination in a particular class is dependent on the number of elements in their respective forward iteration classes.

1. Introduction

Cellular automata, originally developed by von Neuman and Ulam in the 1940's to model biological systems, are discrete dynamical systems that are known for their strong modeling and self-organizational properties (for examples of some modeling properties see [3,5,22–24,26]). Cellular automata are defined on an infinite lattice and can be defined for all dimensions. In the one-dimensional case the integer lattice $\mathbb{Z}$ is used. In the two-dimensional case, $\mathbb{Z} \times \mathbb{Z}$. An example of a two-dimensional cellular automata is John Conway's ever popular "Game of Life"[1]. Probably the most interesting aspect about cellular automata is that which seems to conflict our physical systems. While physical systems tend to maximal entropy, even starting with complete disorder, forward evolution of cellular automata can generate highly organized structure.

As with all dynamical systems, it is important and interesting to understand their long term or evolutionary behavior. Hence it makes sense to develop a classification of a system based on its dynamical behavior. The concept of classifying cellular automata was initiated by Stephen Wolfram in the early 1980's, see [25,26]. Through numerous computer simulations, Wolfram noticed that if an initial configuration (sequence) was chosen at random the probability is high that a cellular automaton rule will fall within one of four classes.

The examples to follow are referred to by a rule numbering system developed by Wolfram, see [25,27]. In [27], one-dimensional cellular automata are partitioned into four classes depending on their dynamical behavior, see Fig. 1 (Totalistic Rule 36) for an example of a Wolfram class 1 cellular automaton. Class 1 are the least chaotic, indeed Wolfram labeled these as automata that evolve to a uniform state. Fig. 2 (Totalistic Rule 24) is an example of a Wolfram class 2 cellular automaton.

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[1] For a complete description (including some of the more interesting structures that emerge) of "The Game of Life" see [1] Chapter 25.

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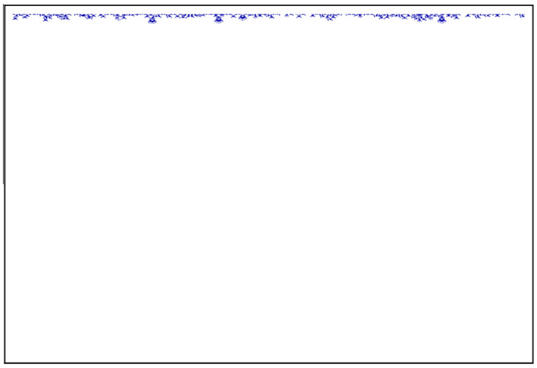

Fig. 1. Wolfram class 1 cellular automaton (Totalistic Rule 36).

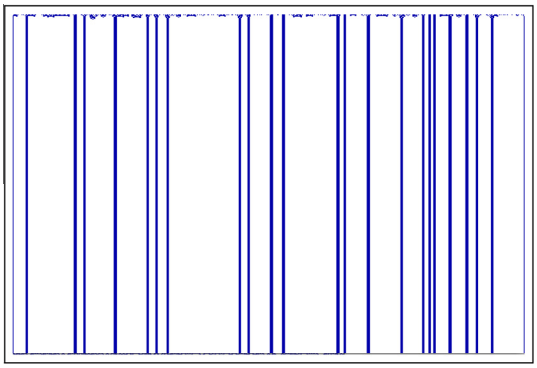

Fig. 2. Wolfram class 2 cellular automaton (Totalistic Rule 24).

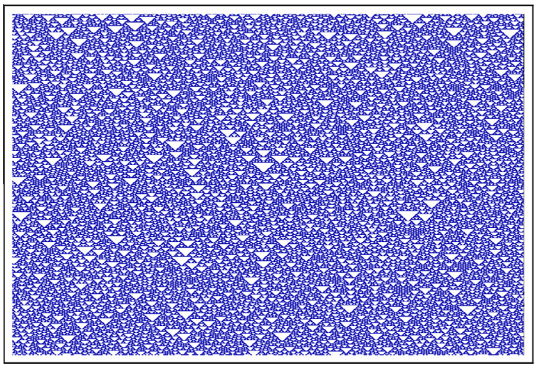

Fig. 3. Wolfram class 3 cellular automaton (Totalistic Rule 12).

Wolfram described the evolution of automata of this class as leading to simple stable or periodic structures. Fig. 3 (Totalistic Rule 12) is an example of a Wolfram class 3 cellular automaton. In these automata the dynamical behavior is more complicated, however triangles and other small structures are seen to emerge in the form of a chaotic pattern. Fig. 4 (Totalistic Rule 20) is an example of a class 4 cellular automaton. Wolfram labeled class 4 the most chaotic whereby localized complex structures emerge. In these figures it can be seen that a cellular automaton map starts with a given (random) initial configuration and evolves in a downward direction upon forward iterations (evolution) of the cellular automaton rule. It is interesting to note the two persisting structures that emerge in the Fig. 4 automaton. The structure on the left evolves straight down, while the structure on the right evolves on a diagonal. Eventually the one on the right will 'crash' into the structure on the left and they will either annihilate each other or produce another persisting structure.

A later and more rigorous classification scheme for one-dimensional cellular automata, see [7], was developed by Robert Gilman. Here a probabilistic/measure theoretic classification scheme was developed based on the probability of choosing a