Applied Mathematics and Computation 258 (2015) 1–11

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Synchronization for complex dynamical networks with time delay and discrete-time information

Mei Fang

Business and Trade School, Ningbo City College of Vocational Technology, Ningbo 315000, China

ARTICLE INFO

Keywords:
Complex dynamical networks
Discrete-time communication
Synchronization
Time delay

ABSTRACT

The synchronization problem is studied for complex dynamical networks (CDNs) with time delay based on discrete-time communication pattern. A new Lyapunov functional is proposed, which is positive definite at communication instants but not necessarily positive definite inside the communication intervals, and can make full use of the available information about the discrete-time communication pattern. Based on the Lyapunov functional, an exponential synchronization criterion is proposed to ensure the synchronization of the considered CDNs. The effectiveness of the developed results are shown by the numerical simulation.

1. Introduction

In the past decades, complex dynamical networks (CDNs) which consist of interacting dynamical entities with an interplay between dynamical states and interaction patterns has been intensively studied by many researchers and numerous results have been reported due to the fact that many systems in nature can be modelled by CNDs, for example, power grids, the Internet, electrical power grids, food webs and the World Wide Web. As a result, a great number of important research results have been published on this topic [2,1,3–14]. For example, in [8], the synchronization problem has been studied for discrete-time delayed complex networks with stochastic nonlinearities, multiple stochastic disturbances, and mixed time delays, and by utilizing the properties of Kronecker product, the free-weighting matrix method, and the stochastic techniques, the synchronization stability criteria have been proposed, which can be readily checked by using standard numerical software. In [9], the synchronization problem has been considered for discrete-time stochastic complex networks over a finite horizon, and based on a time-varying real-valued function and the Kronecker product, the criteria have been established that ensure the bounded H_∞ synchronization. The distributed adaptive control of synchronization in complex networks has been considered in [10], where an effective distributed adaptive strategy to tune the coupling weights of a network has been designed based on local information of node dynamics.

On the other hand, much attention has been drawn to the study of sampled-data control systems because modern control systems usually employ digital technology for controller implementation [15–22]. For example, in [15] the synchronization problem of a complex dynamical network with coupling time-varying delays via delayed sampled-data controller has been investigated and a stability condition has been proposed to find the controller which achieves the synchronization of a complex dynamical network with coupling time-varying delay. In [17], the sampled-data control of linear systems under uncertain sampling with the known upper bound on the sampling intervals has been considered and for the first time, the time-dependent Lyapunov functionals has been proposed to guarantee the stability of systems under the time-varying

E-mail address: fmeicity@163.com

http://dx.doi.org/10.1016/j.amc.2015.01.106

sampling. In [21], a novel approach to assess the stability of continuous linear systems with sampled-data inputs has been given, which had provided easy tractable stability conditions for the continuous-time model, and sufficient conditions for asymptotic and exponential stability have been provided dealing with synchronous and asynchronous samplings and uncertain systems. Recently, the discrete-time communication pattern has been proposed for continuous-time CDNs based on the concept of sampled-data control systems in [23], where a piecewise Lyapunov–Krasovskii functional has been employed to govern the characteristics of the discrete communication instants, and a synchronization criterion has been derived and an upper bound of the communication intervals has been obtained. However, the time delay has not been considered in [23]. It is well-known that time-delay is frequently encountered in a variety of engineering systems [24–36], and a relatively small time-delay may lead to non-synchronization or significantly deteriorated performances for the CDNs. Therefore, it is necessary to study the synchronization for CDNs with *time delay* based on discrete-time communication pattern, which is the motivation for this paper.

The synchronization problem of CDNs with time delay is investigated based on discrete-time communication pattern. A new time-dependent Lyapunov functional is proposed, which make full use of the available information about the discrete-time communication pattern. Based on the Lyapunov functional, an exponential synchronization criterion is proposed to ensure the synchronization of the considered CDNs. A simulation example is given to show the efficiency of the proposed method.

2. Preliminaries

Consider a CDN consisting of N identical nodes coupled in the discrete-time way:

$$\dot{x}_i(t) = Ax_i(t) + B\bar{f}(x_i(t)) + C\bar{f}(x_i(t-d)) + c\sum_{j=1}^{N} g_{ij}\Gamma x_j(t_k),\ t \in [t_k,\ t_{k+1}), \quad i = 1,2,\ldots,N, \tag{1}$$

where $x_i(t) = [\,x_{i1}(t) \quad x_{i2}(t) \quad \ldots \quad x_{in}(t)\,]^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector of node i, $\bar{f} : \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear vector-valued function, and c is a constant denoting the coupling strength. $A, B, C, \Gamma \in \mathbb{R}^{n\times n}$ are constant matrices and Γ denotes the inner-coupling matrix. $G = (g_{ij})_{N\times N}$ is the coupling configuration matrix. If there is a connection from node j and node i $(i \neq j)$, then $g_{ij} > 0$, otherwise, $g_{ij} = 0$ $(i \neq j)$. The diagonal elements of matrix G are defined by

$$g_{ii} = -\sum_{j=1, j\neq i}^{N} g_{ij}. \tag{2}$$

On the other hand, $t_k, k = 0,1,2,\ldots,$ are the communication instants satisfying $0 = t_0 < t_1 < \cdots < t_k < \cdots < \lim_{k\to+\infty} t_k = +\infty$. In this paper, the distance between any two consecutive communication instants is assumed to be bounded. Specifically, it is assumed that $t_{k+1} - t_k = h_k \leqslant h$ for all $k \geqslant 0$, where $h > 0$ represents the largest communication interval.

Let

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix} \in \mathbb{R}^{Nn},\ g(x(t)) = \begin{bmatrix} \bar{f}(x_1(t)) \\ \bar{f}(x_2(t)) \\ \vdots \\ \bar{f}(x_N(t)) \end{bmatrix} \in \mathbb{R}^{Nn},$$

then CDN (1) can be written as

$$\dot{x}(t) = (I_N \otimes A)x(t) + (I_N \otimes B)g(x(t)) + (I_N \otimes C) \times g(x(t-d)) + c(G \otimes \Gamma)x(t_k), \quad t \in [t_k,\ t_{k+1}). \tag{3}$$

Throughout this paper, we make the following assumption on $\bar{f}(\cdot)$.

Assumption 1. For $\forall u, v \in \mathbb{R}^n$, the nonlinear function $\bar{f}(\cdot)$ is continuous and assumed to satisfy the following sector-bounded nonlinearity condition:

$$(\bar{f}(u) - \bar{f}(v) - F_1(u-v))^{\mathrm{T}}(\bar{f}(u) - \bar{f}(v) - F_2(u-v)) \leqslant 0, \tag{4}$$

where F_1 and F_2 are known constant real matrices.

Remark 1. It is noted that such type of description of nonlinear function is known as the sector-like condition, which is originated from [37] and more general than the commonly used Lipschitz conditions. It is not difficult to prove that for any nonlinear function $\bar{f}(\cdot)$ satisfying (4), there exists a scalar $\rho > 0$ such that

$$\|\bar{f}(u) - \bar{f}(v)\|^2 \leqslant \rho\|u - v\|^2. \tag{5}$$