Applied Mathematics and Computation 258 (2015) 120–129

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Binary 3-point and 4-point non-stationary subdivision schemes using hyperbolic function

Shahid S. Siddiqi [a,*], Wardat us Salam [a], Kashif Rehan [b]

[a] Department of Mathematics, University of the Punjab, Quaid-e-Azam Campus, Lahore 54590, Pakistan
[b] Department of Mathematics, University of Engineering and Technology, KSK Campus, Lahore, Pakistan

ARTICLE INFO

Keywords:
Non-stationary subdivision scheme
Binary
Approximating
Hyperbolic functions

ABSTRACT

In this paper, binary 3-point and 4-point non-stationary subdivision schemes are presented using hyperbolic function as basis function. Asymptotic equivalence method is used to investigate the continuity of the proposed schemes. Comparison between the proposed schemes, their stationary counterparts and some existing non-stationary schemes has been depicted through examples. It can be observed that the proposed schemes have the capability to reproduce conics particularly parabola.

1. Introduction

Subdivision schemes provide efficient iterative algorithms aimed at constructing smooth curves and surfaces as well as fractal curves and surfaces [1,2], by recursively refining the initial control polygon. Thats why subdivision schemes are widely used in computer aided geometric design, reverse engineering and signal & image compression *etc.* Subdivision schemes comprise recursive refinements of an initial sparse sequence of data with the use of rules that remain the same in all subdivision steps, are stationary, otherwise non-stationary.

The very first stationary subdivision scheme was presented by de Rahm [3] (in 1956), which gives C^1 continuity. Further, in the field of stationary subdivision schemes, Chaikin [4] (in 1974) presented binary 2-point approximating subdivision scheme which gives C^1 continuity; binary 4-point interpolating subdivision scheme presented by Dyn et al. [5] (in 1987), was the first interpolating subdivision scheme, generating C^1 limiting curves; Hassan and Dodgson [6] (in 2003) developed binary 3-point and ternary 3-point approximating subdivision schemes yielding C^3 and C^2 continuous curves, respectively; Siddiqi and Ahmad [7] (in 2007) proposed binary 3-point approximating subdivision scheme generating C^2 continuous curve; Siddiqi and Rehan [8,9] (in 2010) introduced subdivision schemes based on B-splines. Recently, Siddiqi and Younis [10,11] generalized the binary and ternary subdivision schemes.

Though the stationary subdivision schemes produce smooth curves but, sometimes, it requires to construct subdivision schemes that can reconstruct conics or part of conics. Non-stationary subdivision schemes have the capability to reproduce conic sections.

In the field of non-stationary subdivision schemes; Jena et al. [12] (in 2003) introduced binary 4-point non-stationary subdivision scheme which reproduces a circle. The scheme has been developed using trigonometric Lagrange polynomial, which is the non-stationary counterpart of the scheme [5]; Daniel and Shunmugaraj presented approximating non-station-

* Corresponding author.
E-mail addresses: shahidsiddiqiprof@yahoo.co.uk (S.S. Siddiqi), wardahtussalam@gmail.com (W. Salam), kkashif_99@yahoo.com (K. Rehan).

http://dx.doi.org/10.1016/j.amc.2015.01.091
0096-3003/

ary subdivision schemes [13,14] (in 2008 and 2009) based on trigonometric Lagrange interpolant and trigonometric B-splines; Siddiqi and Younis [15] (in 2012) presented non-stationary subdivision schemes using trigonometric B-splines; Siddiqi and Younis [16] generalized the binary non-stationary subdivision schemes using trigonometric B-spline; Pakeeza Ashraf et al. [17] (in 2014) generalized the binary and quaternary 4-point non-stationary subdivision schemes using trigonometric Lagrange interpolant.

In the following section, some preliminaries are given that are necessary to develop the subsequent results.

2. Background notions

Given a set of initial control points $P^0 = \{P_i^0 \in R^d\}_{i=-1}^{n+1}$, a binary subdivision scheme generates new set of control points $P^k = \{P_i^k\}_{i=-1}^{2^k n+1}$ at level $k (k \geqslant 0, k \in Z)$ by the subdivision rule

$$P_i^{k+1} = \sum_{j \in z} a_{i-2j}^k P_j^k, \quad i \in Z,$$

where the set of coefficients $a^{(k)} = \left\{a_i^{(k)}, i \in Z\right\}$ in above equation is termed as the mask of the subdivision scheme at kth subdivision step. The Laurent polynomial associated with the non-stationary subdivision scheme $\{S_{a_k}\}$ having mask $a^{(k)}$ is

$$a_k(z) = \sum_{i \in Z} a_i^{(k)} z^i, \quad k \geqslant 1.$$

Definition 1. [18] Let a^k be the mask at the kth level of the subdivision scheme $\{S_k\}$. Then the set $\{i \in Z : a_i^k \neq 0\}$ is called the support of the mask a^k.

Definition 2. Two binary subdivision schemes $\{S_{a_k}\}$ and $\{S_{b_k}\}$ are asymptotically equivalent if

$$\sum_{k=1}^{\infty} \|S_{a_k} - S_{b_k}\| < \infty,$$

where $\|S_{a_k}\|_\infty = \max\left\{\sum_{i \in Z}\left|a_{2i}^{(k)}\right|, \sum_{i \in Z}\left|a_{2i+1}^{(k)}\right|\right\}$.

Theorem 1 [19]. *Let $\{S_{a_k}\}$ and $\{S_a\}$ be the two asymptotically equivalent subdivision schemes having finite masks of the same support. Suppose $\{S_{a_k}\}$ is a non-stationary subdivision scheme and $\{S_a\}$ is a stationary subdivision scheme. If $\{S_a\}$ is C^m and $\sum_{k=0}^{\infty} 2^{mk}\|S_{a_k} - S_a\| < \infty$, then the non-stationary subdivision scheme $\{S_{a_k}\}$ is C^m.*

2.1. Hyperbolic function

Suppose a data set

$$D = \{(x_j, f(x_j)) : j = 0, 1, \ldots, n\}.$$

General form of hyperbolic function of D is

$$H(x) = \Sigma_{j=0}^{n} f(x_j) H_j(x),$$

where

$$H_j(x) = \Pi_{k=0,k\neq j}^{n} \cosh\left(\frac{\beta(x - x_j)}{n}\right) \frac{\sinh\left(\frac{\beta(x-x_k)}{n}\right)}{\sinh\left(\frac{\beta(x_j-x_k)}{n}\right)}, \quad \text{when} \quad n = 2m - 1 \tag{1}$$

and

$$H_j(x) = \Pi_{k=0,k\neq j}^{n} \frac{\sinh\left(\frac{\beta(x-x_k)}{n}\right)}{\sinh\left(\frac{\beta(x_j-x_k)}{n}\right)}, \quad \text{when} \quad n = 2m \tag{2}$$

for some $\beta,\ \beta \in \mathbb{R}^+$. Define a space of hyperbolic polynomials as

$$T_n := span\{\cosh(j\beta x), \sinh(j\beta x) : j = 0, 1, 2, \ldots, m\}.$$

It is known that the function $H(x)$ belongs to T_n and interpolates D. It is also known that when n is even $H(x)$ is unique function in T_n which interpolates D and there exist many functions in T_n interpolating D, when n is odd. But $H(x)$ is a unique function in T_n which interpolates D and has the minimum amplitude among the other interpolants from T_n.