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# Stabilization of memory type for a rotating disk–beam system ☆

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ABSTRACT

In this article, a rotating disk–beam system is considered. Specifically, the system consists of a flexible beam and a rigid disk which rotates with a time-varying angular velocity. The beam, free at one end and clamped at the other one to the center of the disk, is supposed to rotate with the disk in another plane perpendicular to that of the disk. To stabilize the system, we propose a feedback law which consists of a control torque applied on the disk, while either a boundary or distributed internal control with **memory** is exerted on the beam. Then, it is shown, in both cases, that the closed-loop system is stabilized under suitable conditions on the angular velocity and the memory terms.



## 1. Introduction

This article is intended to provide a comprehensive treatment of the exponential stabilization, under the presence of memory terms, for the following system

$$\begin{cases} \rho y_{tt}(x,t) + EIy_{xxxx}(x,t) + \mathcal{E}(t) = \rho\omega^2(t)y(x,t), & (x,t) \in (0,\ell)\times(0,\infty), \\ y(0,t) = y_x(0,t) = y_{xx}(\ell,t) = 0, & t > 0, \\ EIy_{xxx}(\ell,t) = \mathcal{F}(t), & t > 0, \\ \frac{d}{dt}\left\{\omega(t)\left(I_d + \rho\int_0^\ell y^2(x,t)dx\right)\right\} = \Gamma(t), & t > 0, \\ y(x,0) = y_0(x), \quad y_t(x,0) = y_1(x), & x \in (0,\ell), \\ \omega(0) = \omega_0 \in \mathbb{R}, \end{cases} \tag{1.1}$$

in which the physical parameters $\ell,\ EI,\ \rho$ and $I_d$ are respectively the length of the beam, the flexural rigidity, the mass per unit length of the beam, and the disk's moment of inertia. Furthermore, $y(x,t)$ is the beam's displacement in the rotating plane at time $t$ with respect to the spatial variable $x$ and $\omega$ is the angular velocity of the disk. Also, $\mathcal{E}(t),\ \mathcal{F}(t)$ and $\Gamma(t)$ denote respectively the internal distributed **memory** type control, the **memory** type boundary control exerted on the beam and the torque control to be applied on the disk.

The above system models the dynamics of a large-scale flexible space structure [3], namely, a flexible robot beam/arm, clamped at one end to a rigid body (disk) and free at the other end. Moreover, it is assumed that the center of mass of the disk is fixed in an inertial frame and rotates in that frame with a nonuniform angular velocity [3].

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The stabilization problem of the system (1.1) has attracted the attention of many researchers (see for instance [3,4,26,20,21,17,10,7,9] and the references therein). To be more precise, the authors showed in [3] that with only a structural damping control

$$\mathcal{E}(t) = r_1 y_{xxxxt}(x,t), \quad r_1 > 0,$$

the system has a finite number of rotating equilibrium states. Also, in the case of a much weaker viscous damping control

$$\mathcal{E}(t) = r_2 y_t(x,t), \quad r_2 > 0,$$

the existence of a flat linear inertial manifold has been showed for the system [4]. However, in the presence of the viscous damping and torque control in (1.1)

$$\begin{cases} \mathcal{E}(t) = k y_t(x,t), \quad k > 0, \\ \Gamma(t) = -d(\omega(t) - \varpi), \quad d > 0, \quad \varpi \in \mathbb{R}, \end{cases}$$

an exponential stability result has been established in [26]. The stabilization problem of variants of the system (1.1) has been dealt with in [20,21]. Indeed, under specific conditions, the author in [20] proved that the following system

$$\begin{cases} \rho y_{tt}(x,t) + EIy_{xxxx}(x,t) + \rho\ddot{\theta}(t)(b+x) - \rho\dot{\theta}^2(t)y(x,t) = 0, & (x,t) \in (0,\ell)\times(0,\infty), \\ y(0,t) = y_x(0,t) = 0, & t > 0, \\ EIy_{xx}(\ell,t) = -k_1 y_{xt}(\ell,t), \quad k_1 > 0, & t > 0, \\ EIy_{xxx}(\ell,t) = k_2 y_t(\ell,t), \quad k_1 > 0, & t > 0, \\ I_d\ddot{\theta}(t) + EI(by_{xxx}(0,t) - y_{xx}(0,t)) = \Gamma(t), & t > 0, \end{cases}$$

can be exponentially or asymptotically stabilized depending on the expression of the torque controls $\Gamma(t)$. Note also that if the angular velocity $\omega$ is assumed to be constant in (1.1), then a linear variant of (1.1), namely,

$$\begin{cases} \rho y_{tt}(x,t) + EIy_{xxxx}(x,t) = \rho\Omega^2 y(x,t), & (x,t) \in (0,\ell)\times(0,\infty), \\ y(0,t) = y_x(0,t) = 0, & t > 0, \\ \omega(t) = \Omega \ (constant), & t > 0, \end{cases}$$

would be uniformly exponential stabilized if dynamic boundary force and moment controls are applied at the free end of the beam [21]

$$\begin{cases} EIy_{xx}(\ell,t) = -k_1 y_{xt}(\ell,t), \quad k_1 > 0, \quad t > 0, \\ EIy_{xxx}(\ell,t) = k_2 y_t(\ell,t), \quad k_1 > 0, \quad t > 0. \end{cases}$$

This result has been extended to the system (1.1) with one boundary control (force or moment), in addition of a control torque of the disk [17]. Later, the mathematical finding in [17] (resp. [21]) has been improved in [8] to the case of dynamic controls (resp. non-homogeneous beam [9]). Last but not least, we point out that even nonlinear controls have been proposed in some works related to the system (1.1). In fact, it has been showed in [10] that the system can be asymptotically stabilized by only a nonlinear feedback torque control law, whereas a class of nonlinear controls has been provided in [7] to ensure the exponential stability of the system.

As the reader has certainly noticed, the feedback controls proposed in literature do not take into account the memory feature which is inevitable in practical control systems (see e.g. [2,6,13,14,18,19,22–24] for other kinds of systems with various types of memory terms). Whence the stability of the system (1.1) with memory terms is worth studying for mathematical as well as practical reasons.

For ease, we shall treat only the case $EI = \rho = \ell = 1$ thanks to a well-known scaling argument (change $y(x,t)$ to $y(x\ell, \sqrt{(\ell^4/EI)\rho t})$). The main contribution of this article is to show that under suitable conditions, the system (1.1) can be exponentially stabilized via either the feedback control law

$$\begin{cases} \mathcal{F}(t) = \alpha y_t(1,t) + \beta \displaystyle\int_{t-\tau_2}^{t-\tau_1} \lambda(t-s) y_t(1,s)ds, \\ \Gamma(t) = -\gamma(\omega(t) - \varpi), \quad \varpi \in \mathbb{R}, \end{cases} \tag{1.2}$$

or

$$\begin{cases} \mathcal{E}(t) = p(x)y_t(x,t) + q(x) \displaystyle\int_{t-\tau_2}^{t-\tau_1} \lambda(t-s) y_t(x,s)ds, \\ \Gamma(t) = -\gamma(\omega(t) - \varpi), \quad \varpi \in \mathbb{R}, \end{cases} \tag{1.3}$$

in which $\alpha > 0$, $\beta \in \mathbb{R}$ and $\gamma > 0$ are feedback gains; $\tau_1, \ \tau_2 \in \mathbb{R}$ such that $0 \leqslant \tau_1 < \tau_2$; $\lambda \in L^\infty(\tau_1, \tau_2)$ is the memory kernel and $p, \ q \in L^\infty(0,1)$.

The novelty of the present work, compared to the previous ones, lies in the fact that we do take into consideration a memory term in (1.2) and (1.3). Notwithstanding, it is shown, by adopting the same strategy as in [23] for a wave equation, that