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2N order compact finite difference scheme with collocation method for solving the generalized Burger's–Huxley and Burger's–Fisher equations

D. A. Hammad [a,*], M. S. El-Azab [b]

[a] Basic Engineering Sciences Department, Benha Faculty of Engineering, Benha University, Benha 13512, Egypt
[b] Mathematics and Engineering Physics Department, Faculty of Engineering, Mansoura University, Mansoura 35516, Egypt

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ABSTRACT

The generalized Burger's–Huxley and Burger's–Fisher equations are solved by fully different numerical scheme. The equations are discretized in time by a new linear approximation scheme and in space by 2N order compact finite difference scheme, after that a collocation method is applied. Also, the two-dimensional unsteady Burger's equation is described by our proposed scheme. Numerical experiments and numerical comparisons are presented to show the efficiency and the accuracy of the proposed scheme.

1. Introduction

Nonlinear partial differential equations (NPDEs) are commonly used to model most phenomena in science and engineering. The generalized Burger's–Huxley equation (GBHE), the generalized Burger's–Fisher equation (GBFE) and two-dimensional unsteady Burger's equation are examples of these equations, which describe the interaction between reaction mechanisms, convection effects and diffusion transports [1]. Obtaining an efficient and more accurate numerical solution for such equations has been the subject of many studies (see [2–6,10,14,15], and references therein).

Our contribution in this paper is to develop a general compact finite difference scheme of order 2N for solving the following nonlinear partial differential equations (NPDEs):

$$\textbf{I-}\ u_t + \mu u^{\delta} u_x - u_{xx} = f(u), \quad (x,t) \in D \times I, \tag{1.1}$$

with the initial condition

$$u(x,0) = G(x), \quad x \in D, \tag{1.2}$$

and the boundary conditions

$$u(a,t) = H_1(t), \quad t \in I, \tag{1.3}$$

$$u(b,t) = H_2(t), \quad t \in I, \tag{1.4}$$

II- The two-dimensional unsteady Burger's equation [14,15]:

* Corresponding author.
E-mail address: doaa.hammad@bhit.bu.edu.eg (D. A. Hammad).

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$$u_t + uu_x + uu_y = \nu(u_{xx} + u_{yy}), \quad (x,y) \in \Omega \times I, \tag{1.5}$$

with the initial condition

$$u(x,y,0) = G_0(x,y), \quad (x,y) \in \Omega, \tag{1.6}$$

and the boundary conditions

$$u(x_0,y,t) = H_3(y,t), \quad u(x_{M_x},y,t) = H_4(y,t), \quad y_0 \leqslant y \leqslant y_{M_y}, \quad t \in I, \tag{1.7}$$

$$u(x,y_0,t) = H_5(x,t), \quad u(x,y_{M_y},t) = H_6(x,t), \quad x_0 \leqslant x \leqslant x_{M_x}, \quad t \in I, \tag{1.8}$$

where $D = \{x : x \in [a,b]\}$, here a and b are constants, $\Omega = \left\{(x,y) : x \in [x_0, x_{M_x}], y \in \left[y_0, y_{M_y}\right]\right\}$, $\nu = \frac{1}{Re} > 0$, Re is the Reynolds number, I is a time interval $[0,T]$ and the subscripts x, y and t denote to the spaces and time derivatives, respectively. Eq. (1.1) is called GBHE if $f(u) = \beta u(1-u^\delta)(u^\delta - \gamma)$ and it is called GBFE if $f(u) = \beta u(1-u^\delta)$, where μ, β, δ and γ are parameters such that: $\beta \geqslant 0$, $\delta > 0$ and $\gamma \in (0,1)$. For $\mu = 0$ and $\delta = 1$, the GBHE is reduced to the Huxley equation [2] which describes the nerve pulse propagation in the nerve fibres and wall motion in liquid crystals [7,8]. For $\beta = 0$ and $\delta = 1$, the NPDE (1.1) is reduced to the Burgers equation [2] which describes the far field of the wave propagation in nonlinear dissipative systems [9].

Many researchers used the compact finite difference (CFD) method for the solution of the NPDEs as in ([10–12], and references therein). The main objective of this work is to obtain $2N$ order CFD scheme in space with a linearization approximation scheme in time, based on the characteristic method. After that we apply the collocation method to obtain a linear system of algebraic equations that can be solved by direct or iterative methods at each subsequent time level to compute the unknown coefficients of the basis function and then the solution is obtained. The linear approximation scheme is obtained by using a backward finite difference method for the time derivative and then approximating the resulting equation by using a Taylor series to deal with the convection term which is discretized explicitly.

The organization of this paper is as follows: In Section 2, a $2N$ order CFD method for the first, the second, the third, the fourth and the fifth derivatives is presented. Section 3 is devoted to describe and analyze a full discretization to the suggested scheme and the collocation method. Section 4 contains some numerical results for solving the two-dimensional unsteady Burger's equation, GBHE and GBFE to support the proposed numerical scheme. Finally, a conclusion is given in Section 5.

2. The compact finite difference method

Before giving the compact difference scheme, we introduce some notations. First, we construct $M+1$ grid points (nodes) in space by subdividing the interval $[a,b]$ by points $x_i = a + ih$ for $0 \leqslant i \leqslant M$ and space step $h = \frac{b-a}{M}$. Secondly, we define $u_i = u(x_i,.)$ and we denote by $u_i', u_i'', u_i^{(n)}$ the finite difference approximation to the first derivative $\left(\frac{\partial u}{\partial x}\right)_i$, to the second derivative $\left(\frac{\partial^2 u}{\partial x^2}\right)_i$ and to the n derivative $\left(\frac{\partial^n u}{\partial x^n}\right)_i$ at the node i, respectively. Thirdly, we define the first and the second forward difference of u at i [13] as

$$\Delta_F u_i = u_{i+1} - u_i, \tag{2.1}$$

$$\Delta_F^2 u_i = u_{i+2} - 2u_{i+1} + u_i, \tag{2.2}$$

in general, any forward difference of u at i can be obtained by the following recurrence relation [13]

$$\Delta_F^n u_i = \Delta_F\left(\Delta_F^{n-1} u_i\right), \qquad n = 1,2,\ldots. \tag{2.3}$$

Similarly, we define the first and the second backward difference of u at i [13] as

$$\Delta_B u_i = u_i - u_{i-1}, \tag{2.4}$$

$$\Delta_B^2 u_i = u_i - 2u_{i-1} + u_{i-2}, \tag{2.5}$$

in general, any backward difference of u at i can be obtained by the following recurrence relation [13]

$$\Delta_B^n u_i = \Delta_B(\Delta_B^{n-1} u_i), \quad n = 1,2,\ldots. \tag{2.6}$$

Finally, we denote the standard forward and backward finite difference operator for the first derivative by

$$\delta_F u_i = \frac{\Delta_F u_i}{h} = \frac{u_{i+1} - u_i}{h}, \tag{2.7}$$

$$\delta_B u_i = \frac{\Delta_B u_i}{h} = \frac{u_i - u_{i-1}}{h}. \tag{2.8}$$