Applied Mathematics and Computation 258 (2015) 465–482

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Stress analysis of symmetric and anti-symmetric discretely stiffened laminated cantilever beams using displacement-potential field

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ARTICLE INFO

Keywords:
Discrete stiffener
Laminated cantilever
Displacement-potential
Finite-difference method

ABSTRACT

The stress field of a laminated cantilever beam is analyzed under the influence of symmetric and anti-symmetric arrangements of discrete stiffeners at the opposing longitudinal surfaces. An efficient finite-difference computational scheme is developed, in which a new displacement potential is introduced to model the problem of laminated composites. Solutions of stresses at different plies of the laminated cantilever are obtained, some of which, especially those around the critical regions of stiffeners are presented. The effectiveness and accuracy of present approach is verified by comparing the results with the corresponding solutions of analytical as well as standard computational methods.

1. Introduction

The mechanics of composite materials has now become a key subject in the field of solid mechanics. Now-a-days the use of laminated composites is found to increase extensively in almost all areas of structural applications, especially in the construction of aerospace as well as marine structures. In these structural components, the use of stiffeners is quite extensive, which are basically designed to meet the specific requirements of local strength or stiffness. A comprehensive and accurate analysis of stresses in stiffened structures is of great practical interest, since only it can lead a designer to an appropriate selection of the structural parameters.

Among the existing mathematical models for stress analysis of boundary-value problems of elasticity, the stress-function and displacement formulation approaches are noticeable [1–4]. However, the difficulties involved in trying to solve practical stress problems using the existing models have been pointed out by several researches [5,6]. The analytical methods of solution could not gain that much popularity in the field of stress analysis of composite structures, mainly because of the inability of dealing with the mixed and variable nature of physical conditions as well as complex boundary shapes, which are, however, very common in case of actual structures. Stress analysis of stiffened composite structures is mainly handled by numerical methods, especially, the finite element method (FEM). The method has received widespread applications in various aspects of structural analysis, especially for composite structures, a few of which are cited in reference [7–10]. Reliable and accurate prediction of critical stresses, especially at the surfaces of engineering structures is of great importance as far as reliable, safe and economic design is concerned. The uncertainties associated with the accurate prediction of surface stresses by the conventional computational approaches have been pointed out by several researches [11–13]. Moreover, the lack of reliable and accurate alternative computational approaches for the analysis of stress problems, specially of laminated

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http://dx.doi.org/10.1016/j.amc.2015.01.085

composites is realized with due importance, particularly when the appropriateness and accuracy of the solutions obtained by conventional approaches are required to be verified. It would be worth mentioning that, for most of the practical problems of solid mechanics, closed-form exact analytical solutions are hardly available in the literature.

The finite-difference method (FDM) is an ideal numerical technique for obtaining direct numerical solutions of differential equations. Introducing a new boundary modeling approach for finite difference applications of the displacement formulation of solid mechanics, Dow et al. [13] solved the problem of a cantilever beam and reported that the accuracy of FDM in reproducing the state of stresses along the bounding surfaces was much higher than the corresponding accuracy of FE analysis. Comparing the performances of available modeling approaches for the analysis of composite beams with partial shear interactions, the FDM was determined to be an adequate numerical tool for describing the composite behavior of beams, and the corresponding FD solutions were shown to be more accurate when compared with the usual eight degree-of-freedom FEM solutions, even when finer discretization was used for the FEM solutions [14]. Recently, analyzing the results of orthotropic composite panels, the superiority of a potential-function based FDM was verified over conventional computational method in predicting stresses, especially at the regions of transition of boundary conditions [15].

Recently, a general potential-function formulation [16] has been proposed for the analysis of anisotropic composite structures. The formulation has then been extended to the problems of orthotropic composite materials and obtained numerical as well as analytical solutions for a number of composite structures [15,17–18]. The structural durability of a composite panel with a discontinuous stiffener was investigated under compressive loading induced by the gradual displacement of an end support, using the composite durability structural analysis (CODSTRAN) computer code [19]. Using a layer-wise shell theory, a modeling approach of discretely stiffened laminated composite plates and cylindrical shells has been reported for the analysis of stress, vibration and buckling [20]. The existing layer-wise displacement theory of laminated plates was also applied to analyze laminated beams, and analytical solutions were obtained for static bending and free vibration of the beam [21]. Serious attempts had hardly been made so far that can provide accurate and reliable information about local stresses at the critical regions of a laminated cantilever under the influence of discrete stiffeners.

In this paper, the displacement-potential formulation is first extended for the problems of symmetric cross-ply laminated composites, wherein the potential of a new variable reduction scheme [16] is used. Based on the formulation, a finite-difference computational scheme is developed for the numerical solution of displacement, strain and stresses in the laminate as well as individual plies. The displacement potential method (DPM) is then applied to investigate the elastic field of a laminated cantilever beam under the influence of symmetric and anti-symmetric arrangements of discrete stiffeners. Some of the practical issues of interest, for example, ply stresses, stacking sequence, stiffening length, etc. are also analyzed and discussed in context of the present composite beam. Finally, in order to check the appropriateness as well as accuracy of the present DPM, results of stiffened as well as un-stiffened cantilevers are compared with the corresponding solutions obtained by standard computational method as well as analytical method where available.

2. Displacement-potential field formulation for laminated composites

The global stress–strain relations for symmetric cross-ply laminated composites can be expressed through the extensional stiffness matrix $[A]$, under plane stress condition, as follows [22]

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{1}{h}\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix}, \tag{1}$$

where, σ_{ij} and ε_{ij} are the components of stress and strain, respectively, and h is the overall thickness of the laminate. The elements of stiffness matrix (A_{ij}) are given by [22]

$$A_{11} = \sum_{k=1}^{n}\bar{Q}_{11}(h_k - h_{k-1}), \quad A_{12} = \sum_{k=1}^{n}\bar{Q}_{12}(h_k - h_{k-1}), \quad A_{22} = \sum_{k=1}^{n}\bar{Q}_{22}(h_k - h_{k-1}),$$

$$A_{66} = \sum_{k=1}^{n}\bar{Q}_{66}(h_k - h_{k-1}),$$

where,

$$\bar{Q}_{11} = Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta,$$

$$\bar{Q}_{12} = Q_{12}\left(\sin^4\theta + \cos^4\theta\right) + (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta,$$

$$\bar{Q}_{22} = Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta,$$