Applied Mathematics and Computation 258 (2015) 520–536

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Unstructured triangular cellular automata for modeling geographic spread

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ARTICLE INFO

Keywords:
Cellular automata
Unstructured triangular grids
Anisotropy

ABSTRACT

In this work we propose the use of a cellular automata defined on unstructured triangular grids to simulate geographic spread. A grid structure of a finite element implementation is adopted to cellular automata computations. This approach allow us to model and simulate with cellular automata on computational domains with complex geometries (polygonal boundaries), it still retains the easy implementation of cellular automata and does not present the anisotropy induced by regular grids. We show a comparison (storage and number of evaluations required) of our approach with the classical cellular automata implementations on regular grids: rectangular, equilateral triangulation and hexagonal. The geographical spread on unstructured triangular grids is presented by defining two simple cellular automata models a binary spread (two states) and a deforestation spread (three states). Using unstructured triangular grids no anisotropy effects induced by grid and neighborhood are presented; circular fronts spread as circular fronts. Moreover, the use of unstructured triangular grids for cellular automata can simplifies the coupling of cellular automata with other numerical techniques such as finite element or finite volume.

1. Introduction

From of the days of Von Neumann and Ulam who for the first time introduced the concept of cellular automata (CA) to the recent publication of Wolfram's book *A New Kind of Science* [33], researchers of diverse disciplines have been attracted by the simplicity of cellular automata. In the last fifty years, cellular automata have been subjected to physical and mathematical analysis, also new and fascinating applications have emerged in natural and social sciences.

Cellular automata's popularity it is due to their simplicity and to the remarkable potential to model complex systems [28,2]. A cellular automaton A is a tuple (d, S, N, f) where d is the dimension of space, S is a finite set of states, N a finite subset of $\mathbb{Z}^d$ is the neighborhood and f: $S^N \to S$ is the *local rule, or transition function*, of the automaton. A configuration of a cellular automaton is a coloring of the space by S, an element of $S^{\mathbb{Z}^d}$. The global rule $G : S^{\mathbb{Z}^d} \to S^{\mathbb{Z}^d}$ of a cellular automaton maps a configuration $c \in S^{\mathbb{Z}^d}$ to the configuration $G(c)$ obtained by applying f uniformly in each cell: for all position $z \in S^{\mathbb{Z}^d}, G(c)(z) = f(c(z + v_1), \ldots, c(z + v_k))$ where $N = \{v_1, \ldots, v_k\}$. A CA can be seen as a simple model of a spatially extended decentralized system built up of several individual components called cells. The communication between constituent cells is limited to local interactions; each individual cell is in a specific state which changes over time depending on the states of its neighbors. The CA have applications that include diverse aspects of science such as: fluid mechanics [6], environment: pollution, wildfires [30]; biological systems: evolution of the species, growth of populations [31], behavior of colonies of

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http://dx.doi.org/10.1016/j.amc.2015.01.116

Table 1
Table for the number of neighbors.

	Neumann	Moore
Equilateral	3,9,18	12,36,72
Unstructured	3,9,21	*
Rectangular	4,12,24	8,24,48
Hexagonal	6,18,36	6,18,36

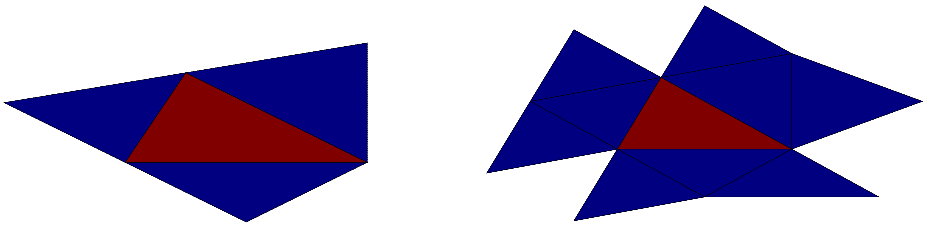

Fig. 1. Left Neumann neighborhood, right Extended Neumann neighborhood.

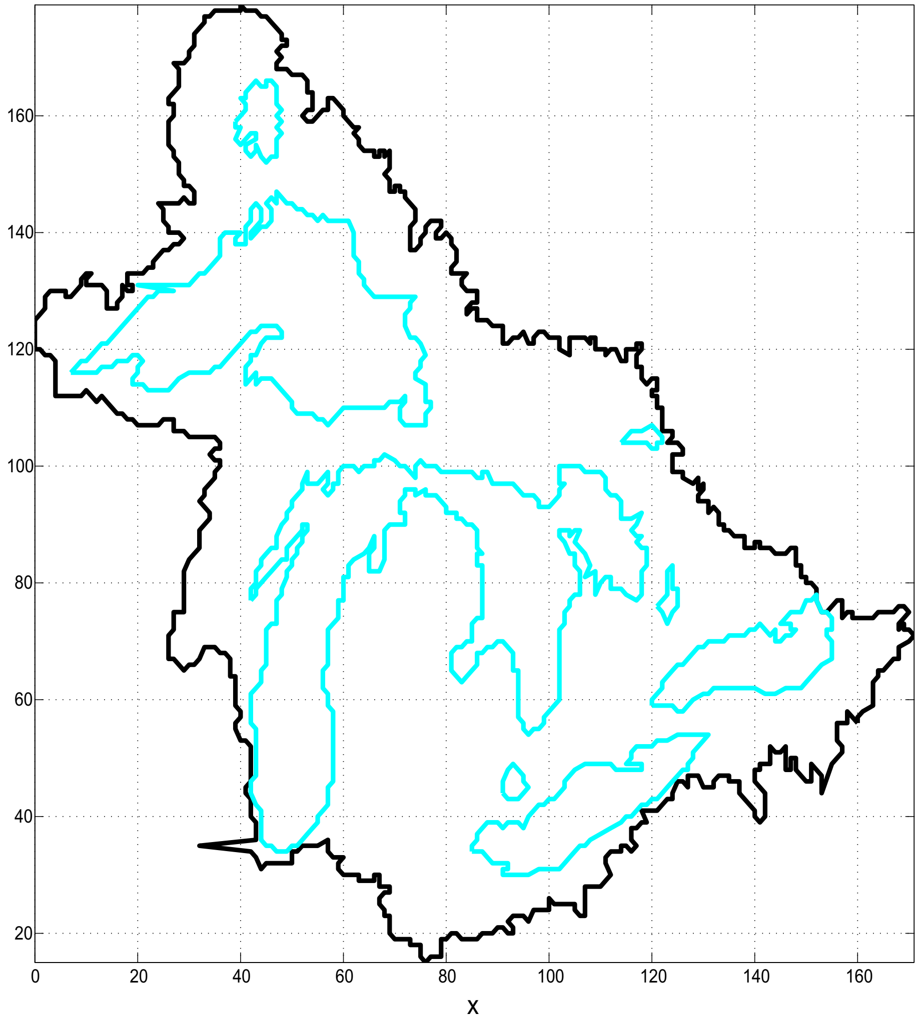

Fig. 2. Great Lakes Basin, a not to scale geographical region bounded by a planar straight line with straight lines (inside) boundaries defining the lakes.