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Unsteady two-layer film flow on a non-uniform rotating disk in presence of uniform transverse magnetic field

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ABSTRACT

Two-layer film flow over a non-uniformly rotating disk in presence of uniform transverse magnetic field is studied numerically under the assumption of planar interface. von *Kármán*'s type similarity variables are used to transform the Navier–Stokes equations into a set of coupled unsteady non-linear partial differential equations. These guiding equations are solved with help of finite difference method. It is found that the lower layer thins faster than the upper layer at the initial stage, later on, upper layer thins faster in all three different type of rotations. Further it is shown that the thinning rate for both the layers slowdown with increasing value of either Ekman or Hartmann number.

1. Introduction

The process of development of thin liquid film over a horizontal rotating disk is known as *spin coating*. In this process, thin film is developed to coat the substrates placed on the grooves of the spinning disk and this technique is widely used in microelectronics industries to coat the photo-resist on silicon wafers for integrated circuits, for magnetic storage disks, etc. The first theoretical study on film flow over rotating disk was carried out by Emslie et al. [1]. In this study, they have considered the balance between the centrifugal and the viscous force during the process of the rotation of disk and they were able to simplify the governing equations and finally able to show that the uniformity of the film is retained as it thins continuously more and more. Washo [2] was the first to examine the applicability of the spin coating process in electronics industry through his experimental as well as theoretical investigations. Meyerhofer [3] introduced the evaporation of the solvent as an important controlling mechanism in spin coating process. Lai [4] has performed several experiments using different solvent and compared his results with Meyerhofer [3]. Chen [5] suggested to include the heat transfer in addition to momentum and mass transfer (solvent evaporation) which were not considered by early researchers. Also, he correlated empirically the film thickness with different physical properties of the fluids. Ma and Hwang [6] have discussed the effect of disk roughness on film thinning. Other researchers have also discussed various physical aspects of the process. In these studies, simplified analysis of Emslie et al. [1] is applied. Higgins [7] first considered the full Navier- Stokes equations and solved analytically for small Reynolds number. Later, Rehg and Higgins [8] solved it numerically for wide range of Reynolds numbers. Dandapat and Ray [9] have discussed the effect of heat transfer from the disk to the film as well as on the development of thin film over a cold rotating disk. Further, Ray and Dandapat [10], Usha and Götz [11] have examined the effect of uniform transverse magnetic field on film thinning

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and found that the magnetic field imparts resistance on single layer film thinning. In these works, it is cleverly considered that the disk is wet, so that the essential no-slip boundary condition can be applied at every point of the disk and the film flows under assumption of planar interface for entire period of spinning.

All the above studies are related with the single layer coating over a surface, but multi-layer coating over a surface is required in many industrial applications (specially, in nuclear industries where radioactive contamination of surface is a big problem) as pointed by Kitamura [12]. To the best of our knowledge, not much attention has been paid to develop a mathematical model for multi-layer film flow over a rotating disk in spite of its important industrial applications (Kitamura [12]). To the best of our knowledge, no experimental work has been reported till to date, but a very few theoretical works were reported in the literature. Sisoev and Shakadov [13] have first studied about the development of two- layer film flow over a rotating disk. After a long gap, McIntyre and Brush [14] have shown that the disturbances to the lower layer have a greater impact on the upper layer than those of disturbances of the upper layer on the lower layer and the disturbances along the upper gas–liquid free surface propagate outward more rapidly than those along the lower liquid–liquid interface. Dandapat and Singh [15] have discussed the effect of viscosity, density and initial film thickness ratio of two different liquids on final film thickness at large time for large value of Reynolds number. Recently, Dandapat and Singh [16] have shown that the viscous force dominates over the centrifugal force and upper layer thins faster than the lower layer at the large time for small values of Reynolds number. Recently, Chen et al. [17] have discussed the importance of spinning disk process in developing thin film. To get better understanding and deeper insight into the transient as well as steady behaviour of the film development over a rotating disk and on thinning of both the layers we are interested to study the effects of three different type of rotations on spin coating process. These different type of rotations are given as follows:

- (i) The angular velocity Ω_0 is set instantaneously (i.e. at time $\tau \to 0_+$) and then maintains its constant speed for the entire subsequent time of rotation.
- (ii) Angular velocity $\Omega = \Omega_0$ accrue instantaneously like (i) and then Ω increases with time *i.e.* $\Omega = \Omega_0(1-\chi\tau)^{-1}$ till $\chi\tau < 1$ is maintained, where $\chi > 0$.
- (iii) The disk starts from rest and its angular speed Ω increases smoothly with time to attain a constant value as $\tau \to \infty$ *i.e.* $\Omega = \Omega_0(1 - \exp^{-\omega\tau})$, where $\omega > 0$.

We are also interested to analyze the effect of uniform transverse magnetic field on thinning of both the layers.

In Section 2, we give a brief description of mathematical problem with corresponding boundary conditions. In Section 3, Numerical procedure for solving the guiding non-linear PDEs is explained. Results and discussions are described in Section 4. Section 5 contains conclusion on the study.

2. Mathematical formulation

Consider a stable system consisting of two immiscible, incompressible, conducting thin uniform liquid films, one laying on top of the other, on a rigid, rotating disk having large diameter. Since diameter of disk is too large in compare to the film thickness, so we can neglect the edge effects. Here, density of lower liquid is greater than upper liquid to give hydrodynamically stable configuration. The lower liquid film which is in contact with the disk is designated as liquid 1, and the liquid on top is designated as liquid 2 (see Fig. 1). The disk is set to rotate with an angular velocity $\Omega(t)$ about the z-axis passing through its center and perpendicular to its plane. B_0 is acting as the external uniform transverse magnetic field, its strength is small enough so that induced magnetic field can be neglected. Following Higgins [7], it is assumed that the initial film thickness of both the layers are uniform and the films remain planar throughout the thinning process. The initial total film thickness is h_{20}, where thickness of lower and upper layers are h_{10} and $h_{20} - h_{10}$, respectively. The axisymmetric form of the governing equations of continuity and motion for incompressible Newtonian fluids are

$$\frac{\partial u_i}{\partial r} + \frac{u_i}{r} + \frac{\partial w_i}{\partial z} = 0, \tag{1}$$

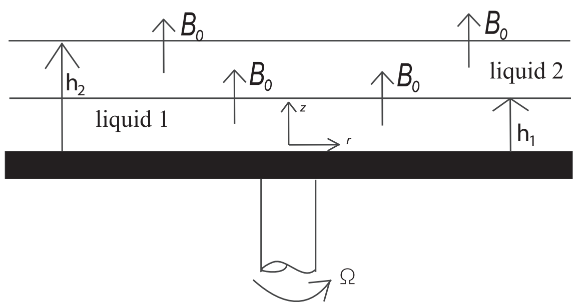

Fig. 1. Schematic diagram.