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Parameter estimation of monomial-exponential sums in one and two variables

L. Fermo *, C. van der Mee, S. Seatzu

Department of Mathematics and Computer Science, University of Cagliari, Viale Merello 92, 09123 Cagliari, Italy

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ABSTRACT

In this paper we propose a matrix-pencil method for the numerical identification of the parameters of monomial-exponential sums in one and two variables. While in the univariate case the proposed method is a variant of that developed by the authors in a preceding paper, the bivariate case is treated for the first time here. In the bivariate case, the method we propose, easily extendible to more variables, reduces the problem to a pair of univariate problems and subsequently to the solution of a linear system. As a result, the relative errors in the univariate and in the bivariate case are almost of the same order.

1. Introduction

In many problems concerning the applied sciences and engineering, it is important to identify the parameters and coefficients $\{n, \{f_j\}_{j=1}^n, \ \{c_j\}_{j=1}^n\}$ in the exponential sums

$$h(x) = \sum_{j=1}^{n} c_j e^{f_j x}, \tag{1}$$

where n is a positive integer, $\{c_j\}_{j=1}^n$ are complex or real coefficients and $\{f_j\}_{j=1}^n$ are distinct complex or real parameters, given a set of $2N$ ($2N \geqslant n$) values of $h(x)$ in equidistant points of $\mathbb{R}$.

This problem arises, in particular, in the propagation of signals [1] [2], electromagnetics [3] [4] and high-resolution imaging of moving targets [5]. The two methods used most are Prony-like (or polynomial) methods and matrix-pencil methods. The first ones are based on the paper by de Prony [6] who was the first to investigate this problem, under the hypothesis that n is known and the data are exact. Several extensions and variants of this method have been proposed to consider the case where n is only approximately known or the data are affected by noise (see, for instance, [7, pp. 458–462], [8–14]). For the matrix-pencil methods, which have been proposed more recently (see, for instance, [15,16]), some attempts to recover the parameters in extended exponential polynomials of the type

$$g(x) = \sum_{j=1}^{n} c_j(x) e^{f_j x},$$

* Corresponding author.
E-mail addresses: fermo@unica.it (L. Fermo), cornelis@krein.unica.it (C. van der Mee), seatzu@unica.it (S. Seatzu).

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where $c_j(x)$ is a polynomial, have been made in particular in [17,18] where no proof of unique reconstruction of the parameters from the data matrix has been given.

More recently, the authors have proposed a matrix-pencil method [19] to estimate the parameters of a monomial-exponential sum of the form

$$h(x) = \sum_{j=1}^{n} \sum_{s=0}^{m_j-1} c_{js} x^s e^{f_j x}, \tag{2}$$

where $\{c_{js}\}_{j=1,s=0}^{n,m_j-1}$ and $\{f_j\}_{j=1}^{n}$ are complex or real parameters and $\{m_j\}_{j=1}^{n}$ are positive integers. In the case $m_1 = m_2 = \cdots = m_n = 1$, the monomial exponential sum $h(x)$, of course, reduces to the exponential sum (1). More precisely, setting

$$M = m_1 + m_2 + \cdots + m_n,$$

the problem is to recover the $M + n$ parameters of h given $2N$ ($N \geqslant M$) observed data. In [19] the uniqueness of the recovery of parameters from the data matrix has been proved.

This problem is of primary interest, for instance, in the direct scattering problem concerning the solution of nonlinear partial differential equations (NPDEs) of integrable type [20] [21].

In this paper we propose a new technique to compute the eigenvalues of the matrix-pencil, that is to identify the parameters $\{f_j\}$ and the order $\{m_j\}$ of the monomials. Our numerical experiments (see Section 4) show that it is as effective as the two techniques proposed in [19], though its computational complexity is lower.

Furthermore we introduce a method to identify the parameters of the following bivariate monomial-exponential sums

$$h(x_1, x_2) = \sum_{j_1=1}^{n_1} \sum_{s_1=0}^{m_{1j_1}-1} \sum_{j_2=1}^{n_2} \sum_{s_2=0}^{m_{2j_2}-1} c_{(j_1,s_1),(j_2,s_2)} x_1^{s_1} e^{f_{1j_1} x_1} x_2^{s_2} e^{f_{2j_2} x_2}, \tag{3}$$

which of course reduces to bivariate exponential sums whenever $m_{1j_1} \equiv m_{2j_2} \equiv 1$ which is the case treated for instance in [22,23]. This method, which reduces the problem to a pair of univariate problems solvable by the method proposed in the univariate case, can easily be extended to more variables.

Let us now outline the organization of the paper. In Section 2 we illustrate our method in the one-variable case and in Section 3 we explain how to treat the bivariate case. Section 4 is devoted to the numerical results and Section 5 to the conclusions.

2. The numerical method for univariate sums

The numerical method we propose to recover the parameters of the monomial-exponential sum (2) reduces the non-linear approximation problem to two problems of linear algebra. The first one is a generalized eigenvalue problem, which allows us to recover n, f_j and m_j. The second one is the solution of a linear system with a Casorati matrix to compute the parameters c_{js}.

Firstly we note that, setting $z_j = e^{f_j} \neq 0$, we can rewrite the monomial exponential sum (2) as a monomial-power sum

$$h(x) = \sum_{j=1}^{n} \sum_{s=0}^{m_j-1} c_{js} x^s z_j^x. \tag{4}$$

For the sake of clarity let us assume initially that $2N$ sampled data with $N \geqslant M$, $M = m_1 + \cdots + m_n$,

$$h(k) = \sum_{j=1}^{n} \sum_{s=0}^{m_j-1} c_{js} k^s z_j^k, \quad 0^0 \equiv 1 \tag{5}$$

are given for the $2N$ integer values $k = k_0, k_0 + 1, \ldots, k_0 + 2N - 1$ with $k_0 \in \mathbb{N}^+ = \{0, 1, 2, \ldots, k_0, \ldots\}$. As we will show in Section 2.3, the problem can be treated as well whenever $h(x)$ is known in $2N$ equidistant points of any interval $[a, b]$. As generally happens in applications, we assume to know a reasonable overestimate $\widehat{M}$ of M and $N \geqslant \widehat{M}$. Under this hypothesis, preliminarily, we arrange the $2N$ given data in the following Hankel matrices of order $N \times \widehat{M}$

$$\mathbf{H}^0_{N\widehat{M}} = \begin{pmatrix} h(k_0) & h(k_0+1) & \ldots & h(k_0+\widehat{M}-1) \\ h(k_0+1) & h(k_0+2) & \ldots & h(k_0+\widehat{M}) \\ \vdots & \vdots & \vdots & \vdots \\ h(k_0+N-1) & h(k_0+N) & \ldots & h(k_0+\widehat{M}+N-2) \end{pmatrix} = \left[\mathbf{h}_0, \mathbf{h}_1, \ldots, \mathbf{h}_{\widehat{M}-1}\right], \tag{6}$$