Applied Mathematics and Computation 258 (2015) 617–630

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Modelling response of phononic Reissner–Mindlin plates using a spectral decomposition

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ARTICLE INFO

Keywords:
Phononic materials
Reissner–Mindlin plate model
Homogenization
Spectral decomposition
Band gaps
Wave dispersion

ABSTRACT

This paper reviews our very recent research focused on modelling of the phononic structures using homogenization of high contrast elastic media. We consider the Reissner–Mindlin type of the homogenized plate which involves frequency-dependent mass coefficients associated with the plate cross-section rotations and the plate deflections. Frequency intervals called band gaps exist for which these coefficients constituting the effective mass matrix can be negative, such that the propagation of elastic waves is restricted, or even stopped. We propose a spectral decomposition based method which is suitable to compute the structure response to an external harmonic loading by forces with frequencies in range of the band gaps. Numerical illustrations are given.

1. Introduction

The phononic plates are periodic structures made of elastic components with large differences in their elastic coefficients; they mimic behaviour of the phononic crystals. Depending on the geometric arrangement of the components it is possible to modulate the plate responses. Modelling of such devices has received an increasing attention (see e.g. [1,2]) for their potential applications in the design of filters, resonators, and waveguides.

This study presents a contribution to the very challenging area of the metamaterial engineering in the context of structural vibration control, where plate and shell structures are frequently used. The phenomenon which is responsible for the band gaps occurrence is related to anti-resonance effects in large contrast elastic media with periodic structures. Homogenization of such structures provides models of equivalent continua with indefinite or negative mass for certain frequencies, as reported in a number of papers [3–6]. Another promising approach which treats such structures in the framework of the generalized continua was followed e.g. in [7] where a relaxed micromorphic continuum model was studied. Such approach is also related to this work since the Reissner–Mindlin plates represent, in a way, a generalized continuum, cf. [8].

To derive the phononic plate model we followed the approach of [4,9]. In [10] we applied the two-scale homogenization to obtain limit plate models. Two cases were studied: 1) according to the Reissner–Mindlin (R–M) theory the plate deformation was described by the mid-plane deflections and by rotations of the plate cross-sections which accounted for the shear stress effects; 2) using the Kirchhoff–Love theory, the plate deflections were described by the bi-harmonic operator, thus

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http://dx.doi.org/10.1016/j.amc.2014.12.037
0096-3003/

neglecting the shear effects. In both cases we assumed such heterogeneities which depend on the mid-plate coordinates only, but do not change with the transversal coordinate. As an example we consider plates with soft cylindrical inclusions embedded in a stiffer matrix. The resulting homogenized plate models contain homogenized mass coefficient matrices, which for some frequencies of applied forces can be negative definite or semi-definite. The wave dispersion of guided plane waves was reported in [11]; we showed existence of band gaps in a guided wave propagation in an infinite homogenized R–M plate. For the K–L plate there is always a propagating wave even though the mass tensor associated with the plate rotations is negative-definite. For standing waves the band gap phenomenon is influenced by boundary conditions.

In our present work we consider the Reissner–Mindlin type phononic plate and focus on the analysis of its responses under the loading by oscillating forces in the frequency ranges of band gaps. We recall that the model has been derived in [12,10] using the homogenization approach.

In Section 3 we propose a spectral decomposition based method for analysis of phononic plate response which takes into account specific boundary conditions. If the mass coefficient matrix $(M_{ij}),\ i,j = 1,2,3$, is indefinite for a given frequency of imposed loading, the spectrum of the matrix is split into positive and negative parts and the plate equations for deflection and two rotations (in the sense of the R–M kinematic ansatz; the in-plane displacement modes are treated separately) are split into two transformed subsystems with positive and negative semi-definite diagonal mass matrices. For these subsystems (one negative and one positive), eigenfunctions ξ_r^+, ξ_s^- and eigenvalues $\mu_r^+,\ \mu_s^-$ are computed, so that two modal bases $\{\xi_r^+\}, \{\xi_s^-\}$ can be introduced in which the plate responses are expressed. It is important that $\{\xi_r^+\}$ and $\{\xi_s^-\}$ are not orthogonal in an energetic product associated with the transformed subsystems. This implies that the two subsystems are coupled, so that computations of solution coefficients c_r^+ and c_s^-, as associated with the two bases, are not straightforward. The real structural response for frequencies in the band gaps is expressed using the sum $\sum_r c_r^+ \xi_r^+$.

Using numerical models we illustrate the spectral decomposition based method proposed in this paper to analyze the plate response to a harmonic loading. The aim is to present different types of band gaps in the context of the mode polarization.

1.1. The Reissner–Mindlin heterogeneous plates

We consider heterogeneous structures associated with a given scale parameter $\varepsilon > 0$, which is the ratio between the characteristic lengths of the microstructure and the wave length which is also proportional to the macroscopic lengths. In Section 2 we report the homogenization limit result obtained for $\varepsilon \to 0$ in our work [10]. In what follows the superscript ε is employed to indicate the dependence of oscillating functions on the scale.

We assume the plate Ω is constituted by the matrix Ω_m^ε which must be a connected domain, and by periodically distributed inclusions; their collection forms a domain Ω_c^ε. Thus, $\Omega = \Omega_m^\varepsilon \cup \Omega_c^\varepsilon \cup \Gamma^\varepsilon$, where $\Gamma^\varepsilon = \overline{\Omega_m^\varepsilon} \cap \overline{\Omega_c^\varepsilon}$. Inclusions are disconnected, having the size $\approx \varepsilon$.

The microstructure is generated as a periodic lattice using the representative periodic cell (RPC) denoted by Y. For simplicity, we consider a rectangular RPC with the following definition: $Y = \Pi_{i=1}^2]0,1[\subset \mathbb{R}^2$. The domain Y is decomposed in accordance with Ω into the matrix part Y_m and the inclusion Y_c, so that $\overline{Y_c} \subset Y$ is strictly contained in Y and $Y_m = Y \setminus \overline{Y_c}$.

Material parameters $\mathbb{C}^\varepsilon = (C_{ijkl}^\varepsilon)$, the plane stress elasticity tensor, and γ^ε, the shear stiffness coefficient, vary periodically through Ω. For the sake of simplicity we shall consider only piece-wise constant isotropic material. To retain the phononic effect in the homogenized plate, following the analogous approach employed in the case of phononic 3D periodic structures, we introduce the scaling of the material coefficients in the inclusions:

$$\begin{aligned}
\mathbb{C}^\varepsilon(x) &= \chi_c^\varepsilon(x)\varepsilon^2\mathbb{C}^c + \chi_m^\varepsilon(x)\mathbb{C}^m, \\
\gamma^\varepsilon(x) &= \chi_c^\varepsilon(x)\varepsilon^2\gamma^c + \chi_m^\varepsilon(x)\gamma^m.
\end{aligned} \tag{1}$$

The standard properties (assuming positive Lamé constants) of constant tensors $\mathbb{C}^c,\ \mathbb{C}^m$ and coefficients $\gamma^c,\ \gamma^m$ are assumed.

The density of the two materials is assumed to be of the same order of magnitude, therefore we shall consider $\rho^\varepsilon(x) = \chi_c^\varepsilon(x)\rho^c + \chi_m^\varepsilon(x)\rho^m,\ \underline{\rho} \leqslant \rho^s \leqslant \overline{\rho},\ s = m,c$ where $\underline{\rho},\ \overline{\rho}$ are given positive real numbers.

According to the definition (1), for a real material with a given size of the microstructure and the size of the plate, the scale parameter $\varepsilon_0 > 0$ can be obtained. The stiffness constants $\mathbb{C}^{\varepsilon_0} = \varepsilon_0^2\mathbb{C}_c$ and $\gamma^{\varepsilon_0} = \varepsilon_0^2\gamma_c$ for positions in the inclusions are defined by realistic "physical" values which then yield $\mathbb{C}_c$ and γ_c, by the direct consequence. The limit model obtained by the homogenization, i.e. for $\varepsilon \to 0$, provides an approximation of a real material behaviour; its quality depends on the real contrast $r = |\mathbb{C}_c^{\varepsilon_0}|/|\mathbb{C}_m| \approx |\gamma_c^{\varepsilon_0}|/|\gamma_m|$. In [9] we demonstrated on numerical examples with 2D phononic structures, how this contrast between the two materials influences the accuracy of modelling the wave dispersion by the homogenized model.

1.2. The Reissner–Mindlin plate model

The plate model can be derived by an asymptotic analysis of the three-dimensional elasticity problem imposed in $\Omega \times]-h,h[$, where $\Omega \subset \mathbb{R}^2$ is an open bounded domain with regular boundary $\partial\Omega$ and h is the plate thickness. In the time interval $[0,T]$ the plate undergoes the following two modes of displacements: the in-plane "membrane modes" described