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Explicit exact solutions of some nonlinear evolution equations with their geometric interpretations

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ABSTRACT

In this paper, the simplest equation method is applied to obtain multiple explicit exact solutions of the combined dispersion equation, the Hirota–Satsuma Korteweg–de Vries system and the generalized Burgers–Huxley equation. These solutions are constructed on the basis of solutions of Bernoulli equation which is used as simplest equation. It is shown that this method is very powerful tool for obtaining exact solutions of a large class of nonlinear partial differential equations. The geometric interpretation for some of these solutions are introduced.

1. Introduction

In the last years construction of exact solutions, in particular, solitary wave solutions of nonlinear partial differential equations (NLPDEs) is an important significance of nonlinear science. Recently, a number of methods for finding exact solutions of NLPDEs, have been proposed such as, the inverse scattering transform [1], the Bӑcklund transform [2,3], the extended tanh- function method [4], the Jacobi elliptic function method [5], the F-expansion method [6,7], the (G'/G)-expansion method [8]. Using the truncated expansion method, exact solitary wave solutions to some nonlinear evolution equations have been obtained [9,10]. In 1988 Kudryashov introduced the simplest equation method [11], for finding exact solutions of nonintegrable nonlinear partial differential equations, and applied by the authors in [12,13]. In [14] Ryabov et al. applied this method for three families of nonlinear evolution equations of fifth, sixth and seventh order. In [15] Kabir et al. were proposed a modified Kudryashov method to construct the solitary traveling wave solutions of the Kuramoto–Sivashinsky equation and seventh-order Sawada–Kotera equation. In fact Kudryashov based this method by taking the auxiliary ordinary differential equation of the form $Q_z = Q^2 - Q$ which contains an exponential function in solution. Vitanov et al. [16] introduced a new approach by taking full advantage of the Bernoulli equation $\psi_z = a\psi^R + b\psi$ and applied by the authors [17–20] to obtain traveling wave solutions of many NLPDEs. When the parameters a, b and R are taken as special values, some solitary wave solutions are derived in terms of hyperbolic functions.

Based on the works as above solution, we apply the simplest equation method to obtain the exact solutions of the combined dispersion equation, the Hirota–Satsuma Korteweg–de Vries (KdV) system and the generalized Burgers–Huxley equation.

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The rest of this paper is organized as follows: In Sections 2, we give brief descriptions of the simplest equation method and the geometric interpretation. In Sections 3–5, we construct exact solutions of the combined dispersion equation, the Hirota–Satsuma KdV system and the generalized Burgers–Huxley equation, respectively. In the last Section, we summarize and discuss our results.

2. Description of method

In this Section, we briefly describe the simplest equation method [11,13,16–19]. The main steps are summarized in the following steps:

For a given NLPDE in the form

$$F(u, u_t, u_x, u_{tt}, u_{xx}, \ldots) = 0. \tag{2.1}$$

In general, the left hand side of Eq. (2.1) is a polynomial in u and its various derivatives.

1. We seek the traveling wave solution of (2.1) in the form

$$u(x,t) = y(z); \quad z = kx - \omega t, \tag{2.2}$$

   where k and ω are constants to be determined later. Under the transformation (2.2), Eq. (2.1) is reduced to an ordinary differential equation (ODE)

$$G(y, y_z, y_{zz}, \ldots) = 0. \tag{2.3}$$

2. The simplest equation method gives the solution of Eq. (2.3) in the form

$$u(x,t) = y(z) = \sum_{j=0}^{N} a_j[\phi(z)]^j, \quad a_N \neq 0, \tag{2.4}$$

   where N in Eq. (2.4) is a positive integer that can be determined by balancing the nonlinear term(s) with the highest derivative term in (2.3), a_j are constants to be determined, and the function $\phi(z)$ takes the form

$$\phi = \left(\frac{1}{1 + e^{(R-1)z+z_0}}\right)^{\frac{1}{R-1}}, \tag{2.5}$$

   where z_0 is a constant of integration. This function is the solution of the auxiliary ordinary differential equation

$$\phi_z = \phi^R - \phi, \tag{2.6}$$

   where R is a positive integer and $R \neq 1$, we will take the constant R in (2.6) by various values to obtain many new and more solutions of (2.3). By using (2.6) repeatedly, all derivatives of y can be expressed in terms of ϕ as

$$y_z = \sum_{j=0}^{N} j a_j \phi^j [\phi^{R-1} - 1],$$

$$y_{zz} = \sum_{j=0}^{N} j a_j \phi^j (\phi^{R-1} - 1)[(j + R - 1)\phi^{R-1} - j],$$

$$y_{zzz} = \sum_{j=0}^{N} j a_j \phi^j (\phi^{R-1} - 1)\left[(j + R - 1)(j + 2R - 2)\phi^{2R-2} - \left((j + R - 1)^2 + j(R - 1) + j^2\right)\phi^{R-1} + j^2\right]. \tag{2.7}$$

   The other derivatives of function y can be calculate in similar way.
3. Substituting (2.4) with (2.7) into the ODE (2.3) and setting each coefficients of the different powers of $\phi(z)$ to zero, we obtain a system of algebraic equations for $a_j\,(j = 0, 1, 2, 3, \ldots, N)$ and the parameters κ, ω.
4. Solving the system for $a_j\,(j = 0, 1, 2, 3, \ldots, N)$, κ and ω by use of Maple or Mathematica. Substituting the obtained coefficients into (2.4), then concentration formulas of traveling wave solutions of the NLPDE (2.1) can be obtained.

- *Remark 2.1.* In [11,13]. Kudryashov determined the value of N in formula (2.4) used the pole order of general solution for Eq. (2.3). By substituting $y(z) = z^{-p}$, where $p > 0$ into all monomials of Eq. (2.3) and comparing the two or more terms with smallest powers in equation then find the value of N. In this paper, we use the balancing to determine the value of N which enable us to construct several types of the solitary wave solutions of Eq. (2.3).

In order to describe the geometric interpretation for the solution of (2.1), we write the solution of (2.1) at the regular regions (there is no singularities) in the form