Applied Mathematics and Computation 251 (2015) 1–13

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

FX options pricing in logarithmic mean-reversion jump-diffusion model with stochastic volatility ☆

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ARTICLE INFO

Keywords:
FX options
Mean reversion
Jump
Stochastic volatility
Attari formula

ABSTRACT

As a tradable asset, foreign currency has the particular property of mean-reversion, which should be reasonably included in FX dynamic modeling. From observation of FX historical data, jump takes frequently and it should be considered as modeling factor as well. The implied volatility smile/skew in FX options market is very significant, thus stochastic volatility is necessary in FX options models. Combining the three factors together, a new model named logarithmic mean-reversion jump-diffusion model with stochastic volatility is constructed. Conditional characteristic function under this model is derived by expectation approach, and Attari's pricing formula is further attained. At last, we give some empirical results to show the good performance of our model.

1. Introduction

As a tradable asset, Foreign Exchange (FX) has the characteristic of mean-reversion, i.e. FX usually oscillates within a certain range and returns to a reasonable level whenever it is too high or too low. The mean-reverting nature of the real and nominal exchange rates is verified by Jorion and Sweeney [11] and Sweeney [17] through empirical analysis. Sorensen [16] assumes mean-reversion dynamics for both foreign and domestic interest rates, by this means they catch the mean-reversion of FX indirectly. Ekvall et al. [4] link the speed of mean-reversion to the degree of central bank intervention. Hui and Lo [10] model FX as a mean-reversion jump-diffusion process and Wong and Lo [18] generalize their model by adding stochastic volatility. The model in this paper is based on the logarithmic mean-reversion and we further consider stochastic volatility and jumps.

Garman and Kohlhagen [7] are the first ones to derive the FX European option price formula using the Black–Scholes approach taking into account the domestic–foreign interest rate differential, and they base on the assumption on the lognormality of the value of the underlying asset with constant volatility. If all B–S assumptions would hold, the implied volatility would be the same for all vanilla options on a specific underlying FX rate. However, in most of the time, the model implied volatilities of options with different maturities and strikes are not constant and tend to be smile/skew shaped. In Merton [15], the price of the underlying asset is assumed to follow a jump-diffusion process, which is obviously more reasonable. The volatility is then allowed to be driven by a stochastic process. The Heston's model proposed in Heston [9] stands out for successfully capturing some major features of volatility observed in the markets, such as mean-reverting

☆ This work is supported by National Natural Science Foundation of China (Nos. 11171304 and 71371168) and Zhejiang Provincial Natural Science Foundation of China (No. Y6110023).

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http://dx.doi.org/10.1016/j.amc.2014.11.040
0096-3003/

and non-negative feature. In addition, it has an analytic characteristic function which can apply Fourier transform techniques. Bates [2] generalizes both Merton's and Heston's model by adding Merton's IID jumps to the stochastic process of the underlying. At short maturities, the desired U-shaped volatility profile is generated by jumps, while at longer maturities, it is generated by stochastic volatilities.

Heston [9] not only firstly introduces stochastic volatility into option modeling, but also a pioneer to employ the characteristic function to option pricing. Following Heston's [9] work, characteristic function method has been applied to other option models. The pricing formula in Heston [9] is based on a guess of solution form by analogy with Black–Scholes formula. The derivation of Heston formula is reduced to solving two PDEs that are virtually similar to the Heston pricing PDE but with simpler initial conditions. The two PDEs can be solved by Fourier transform, and the solutions are expressed by two Fourier inversions which can be implemented by direct numerical integration (DI). Carr and Madan [3] refines the formula form of Heston [9] so that the powerful computation tool of fast Fourier transform can be applied in computing large number of options. Other work can be found in Eraker et al. [6], Eraker [5], Kou and Wang [12], Gatheral [8], Lewis [13] and Lutz [14]. The main motivation of Carr and Madan [3] is to accelerate the computation of the characteristic function represented option pricing formulae. However, the numerical evidence in Kilin and discussion in Zhu [19] show that application of FFT in option pricing may not be so efficient as one may expect. Firstly, a large number of log-strike grids must be used when applying FFT so as to ensure accuracy. This results in large sacrifice of computing efficiency, while only a small number of these option prices are meaningful because most of the strikes are either extremely small or extremely large. Secondly, the options one want to price may hardly fall on the log-strike grids, thus interpolation has to be carried out, which causes further inaccuracy beyond the calculation of grid prices by FFT. In contrast to FFT approach, direct integration by Gauss quadrature only needs dozens of grids for computing one option price. When the number of options need to be computed is not so large, as in most cases in practice, direct integration can still save a lot of time even though it only gets one price in one integration. Furthermore, the 'Multi-domain integration' and 'Strike vector computation' techniques suggested in Zhu [19] can ensure the pricing accuracy while saving plenty of time, because the most time-consuming procedure in our pricing formula is computing characteristic function. By using 'Strike vector computation' technique, we save the characteristic function value for a specific maturity and used repeatedly across all strikes. For the case of this paper, we find direct integration approach is more suitable and is accurate enough for our purpose.

Another simplification to Heston's formula made in Attari [1] reduces the number of integrations from two to one, and obtains a much faster converged integrand than Heston [9]. In this paper we will use the method in Zhu [19] to compute the complex characteristic function, after that we will give the Attari-type price formula and delta formula for vanilla FX options.

The remaining sections of this paper are organized as follows. Empirical study is performed in Section 2 in order to find a suitable model for the FX dynamic. Section 3 proposes the SVJLN model and presents solution for the conditional characteristic function with detailed derivation. Section 4 presents the price formula and delta formula for FX options by using characteristic function. Section 5 conducts a calibration of the pricing formula to FX option quotes and makes comparative study on the model's flexibility of generating volatility smile/skew and their computing speed. The paper is concluded in Section 6.

2. Empirical performances

Fig. 1 shows the EURUSD historical data from January 1, 2007 to December 31, 2013 of EURUSD obtained from the Chinese Wind Financial Database. Firstly, we can find a significant mean-reversion character of the EURUSD: the Euro maintained a distinct appreciation trend relative to Dollar from January 1, 2007 to June, 2008. After that, Euro exchange rate dropped dramatically until November, 2008. In the next few years during that time, the Euro exchange rate experienced numbers of appreciation and depreciation. For this reason, we think it is very important to include mean-reversion as a factor into our model. Moreover, we can obviously find jumps in the historical data in Fig. 1, sometimes the exchange rate jumps up, and sometimes down. Therefore, jumps also need to be added into our model.

Several models are proposed in the literature to account for the characteristic of Foreign Exchange under objective measure P, including the square root model (SR), log-normal model(LN), logarithmic mean-reversion model (LR), and the logarithmic mean-reversion model with jumps (LRJ). We will make an empirical comparison of the four models on fitting EURUSD historical data. These four models for the EURUSD under objective measure P are illustrated in Table 1. For the square-root process, κ, θ and σ are constrained by $2\kappa\theta > \sigma^2$ so as to ensure positivity of FX. W_t is a standard Brownian motion that is independent from the Poisson jump N_t, whose constant intensity is denoted by λ. Therefore, at any time, the FX has probability λdt to jump in the infinitesimal time interval and probability 1-λdt not to jump. The random jump size is assumed to be a double exponential variable $Exp(\eta_1, \eta_2, p)$, with $\eta_1 > 1, \eta_2 > 0$, and $0 < p < 1$, which means both upward and downward jumps are allowed in FX and lnFX. This assumption is necessary if we recall the analysis at the beginning of this section. As displayed in Fig. 1, sudden shocks can drive FX to jump up as well as jump down. There are some other issues that we have taken into consideration. Because the jump rate is constant and the jump size is homogeneous, the poisson compensator is linear function of time, so when we model FX dynamic under the risk–neutral measure Q, we absorb the compensator of jump risk into parameters κ and θ, this means that the dynamics under risk–neutral measure Q will be just the same as that under the objective measure Q, as shown in Table 1.