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Finite-time stability of a class of non-autonomous neural networks with heterogeneous proportional delays

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ABSTRACT

In this paper, the problem of finite-time stability analysis for a class of non-autonomous neural networks with heterogeneous proportional delays is considered. By introducing a novel constructive approach, we derive new explicit conditions in terms of matrix inequalities ensuring that the state trajectories of the system do not exceed a certain threshold over a pre-specified finite time interval. As a result, we also obtain conditions for the power-rate global stability of the system. Numerical examples are given to demonstrate the effectiveness and less restrictiveness of the results obtained in this paper.

1. Introduction

In recent years, dynamical neural networks have received a considerable attention due to their potential applications in many fields such as image and signal processing, pattern recognition, associative memory, parallel computing, solving optimization problems, and so on [1–4]. In most of the practical applications, it is of prime importance to ensure that the designed neural networks be stable. To this point, we refer the reader to a recent work [5]. On the other hand, time delays unavoidably exist in most application networks and often become a source of oscillation, divergence, instability or bad performance [6,7]. A great deal of effort from researchers has been devoted to study the problems of stability analysis, control and estimation for delayed neural networks during the past decade, see, [8–23] and the references therein.

It is well-known that, in practical implementation of neural networks, time delays may not be constants. They are not only time-varying but also proportional in many models [24,25]. Furthermore, as discussed in [26], a neural network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths, it is desirable to model them by introducing continuously proportional delay over a certain duration of time. Proportional delay is one of time-varying (monotonically increasing) and unbounded delays which is different from most other types of delay such as time-varying bounded delays, bounded and/or unbounded distributed delays. Its presence leads to an advantage is that the network's running time can be controlled based on the maximum delay allowed by the network [27]. In addition, dealing with the dynamic behavior of neural networks with proportional delays is an interesting problem which is also much more complicated [26]. So far, there have been few papers in the literature concern with asymptotic behavior and stability of neural networks with proportional delays, see, [26–29] and the references within. Particularly, in [26], by using an exponential transformation in time scale to transform the original model to a model with constant delays, some delay-dependent exponential stability conditions were derived in terms of the spectral radius of a Metzler matrix. However, this approach usually

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leads to conditions that inherit conservatism and to be hard to test by computational tools. By using a similar transformation in combining with the Lyapunov–Krasovskii functional method, some new delay-independent conditions were established in [27] in terms of specific linear matrix inequalities (LMIs) ensuring the global asymptotic stability of a class of neural networks with constant coefficients and single proportional delay. These conditions were improved in a very recent work [29].

However, the aforementioned works have been devoted to neural networks with constant coefficients. On the other hand, it is worth noting that (i) non-autonomous phenomena often occur in many realistic systems, for instance, when considering a long-term dynamical behavior of the system, the parameters of the system usually change along with time [22,30–33]; (ii) stability analysis for non-autonomous systems usually requires specific and quite different tools from the autonomous ones (systems with constant coefficients); and (iii) the approaches proposed in the aforementioned works, for example, [26–29], are not applicable to non-autonomous neural networks with heterogeneous proportional delays, and thus, an alternative approach is obviously needed for the problem of stability analysis for non-autonomous neural networks with proportional delays.

Besides, it is worth to mention here that, while the concept of Lyapunov stability, recognized as infinite-time behavior of such a system, has been well investigated and developed, the concept of finite-time stability (FTS) (or short-time stability) has been extensively studied in recent years, see, [34,35] and the references therein. Roughly speaking, a system is finite-time stable if, for a given bound on the initial condition, its state trajectories do not exceed a certain threshold during a pre-specified time interval. Although the Lyapunov asymptotic stability (LAS) has been successfully applied in many models, FTS is an useful concept to study in many practical systems in the vivid world [34–39]. Furthermore, it is noted that, FTS and LAS are different concepts by mean a system may be finite-time stable but not Lyapunov asymptotic stable and vice versa [35]. Therefore, the problem of finite-time stability analysis for neural networks with delays, especially proportional delays, is of interest and important, and thus, it should be received a greater focus. Looking at the literature in the field of stability analysis for time-delay systems, so far, there has been no result concerned with finite-time stability of non-autonomous neural networks with proportional delay which motivates the present study.

Motivated by the above discussions, in this paper, we first consider the problem of finite-time stability analysis for a class of non-autonomous neural networks with heterogeneous proportional delays. By using a novel approach, based on the idea of comparison techniques which were recently developed in [35,40], an explicit criterion is derived in terms of inequalities for M-matrix ensuring that, for each given bound on the initial conditions, the state trajectories of the system do not exceed a certain threshold over a pre-specified finite time interval. These conditions are shown to be relaxed for the Lyapunov asymptotic stability through examples.

The remainder of this paper is organized as follows. Section 2 presents notations, definitions and some auxiliary results which will be used in the proof of our main results. In Section 3, new explicit conditions for finite-time stability of the system are derived in terms of inequalities for M-matrix. A new power-rate global stability criterion is also presented in this section. Some comparisons to the existing results and illustrative examples are given in Section 4. The paper ends with a conclusion and cited references.

2. Preliminaries

Notations: We let $\mathbb{R}, \mathbb{N}$ denote the set of real numbers and positive integer numbers, respectively. For a given $n \in \mathbb{N}$, we denote $\underline{n} := \{1, 2, \ldots, n\}$. Let $\mathbb{R}^n$ denote the n-dimensional vector space endowed with the norm $\|x\|_\infty = \max_{i\in\underline{n}}|x_i|$ for $x = (x_i) \in \mathbb{R}^n$. The set of real $m \times n$-matrices is denoted by $\mathbb{R}^{m\times n}$. Comparison between vectors will be understood componentwise. Specifically, for $u = (u_i)$ and $v = (v_i)$ in $\mathbb{R}^n$, write $u \succeq v$ $(u \preceq v)$ if $u_i \geqslant v_i$ $(u_i \leqslant v_i)$ for all $i \in \underline{n}$ and $u \succ v$ $(u \prec v)$ if $u_i > v_i$ $(u_i < v_i)$ for all $i \in \underline{n}$. For a given vector $\xi \in \mathbb{R}^n,\ \xi \succ 0$, we denote $\xi^u = \max_{i\in\underline{n}}\xi_i$ and $\xi_l = \min_{i\in\underline{n}}\xi_i$. For a continuous real-valued function $v(t)$, let $D^+v(t)$ denote the upper-right Dini derivative of $v(t)$ defined by $D^+v(t) = \limsup_{h\to 0^+}\frac{v(t+h)-v(t)}{h}$.

Consider a class of non-autonomous neural networks with heterogeneous proportional delays of the following form:

$$\begin{aligned} \dot{x}_i(t) &= -d_i(t)x_i(t) + \sum_{j=1}^{n} w_{ij}^0(t)f_j(x_j(t)) + \sum_{j=1}^{n} w_{ij}^1(t)g_j(x_j(q_{ij}t)) + I_i(t), \quad t > 0, \\ x_i(0) &= x_i^0,\ i \in \underline{n}, \end{aligned} \tag{2.1}$$

where n is a positive integer, $x_i(t)$ is the state variable (potential or voltage) of the ith neuron at time t, $f_j(\cdot),\ g_j(\cdot),\ j \in \underline{n}$, are activation functions, $d_i(t)$ are self-inhibition terms, $w_{ij}^0(t),\ w_{ij}^1(t)$ are time-varying connection weights, $I_i(t)$ are external inputs, $q_{ij} \in (0,1],\ i,j \in \underline{n}$, are possibly heterogeneous proportional delays, $x^0 = (x_1^0, \ldots, x_n^0)^T \in \mathbb{R}^n$, where $x_i^0,\ i \in \underline{n}$, is the initial value of $x_i(t)$ at time $t_0 = 0$.

In this paper, we develop the concept of finite-time stability (FTS) for system (2.1). Roughly speaking, a solution $x^*(t)$ of (2.1) is finite-time stable (for short, we also write as FTS) if, for given a bound on its initial condition, all state trajectories of (2.1) with initial conditions in that bound do not exceed a certain threshold during a specified time interval. System (2.1) is finite-time stable if all solutions of (2.1) are FTS. More precisely, we have the following definition.