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A deceleration model for bicycle peloton dynamics and group sorting

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ABSTRACT

Extending earlier computer models of bicycle peloton dynamics, we add a deceleration parameter by which deceleration magnitude varies as a function of cyclist strength. This model is validated by applying speed data from a mass-start race composed of 14 cyclists, and running simulation trials using 14 simulated cyclists that generated positional profiles which compare well with the positional profiles observed in the actual mass-start race data. Keeping constant the speed variation profile from the mass-start race as introduced into the simulation, a set of simulation experiments were run, including: varying the number of cyclists; varying the duration of a single near-threshold output event; and varying the course elevation. The results consistently show sorting of pelotons into smaller groups whose mean fitness corresponds with relative group position, i.e. fitter groups are closer to the front. Sorting of pelotons into fitness-related groups provides insight into the mechanics of similar group divisions within biological collectives in which members present heterogeneous physiological fitness capacities.

1. Introduction

Pelotons are groups of cyclists coupled by power-output reduction (energy-savings) benefits of drafting. Pelotons may include as many as 200 cyclists, as observed in mass-start bicycle races such as the Tour de France [1].

There is extensive academic literature related to bicycle racing as a sport. This literature can be grouped into three broad categories: bicycle engineering and design, racing strategy, cyclists' physiology and training methods. See [2] for several recent examples. Research involving cycling as a sport, however, generally does not involve quantitative or substantive qualitative analysis of the collective dynamics of pelotons.

The analysis of cyclists' collective dynamics may be traced to the research of Kyle [3] in 1979, which examined the benefits of drafting, thus setting the theoretical basis for the coupled behavior of cyclists. Research involving drafting was subsequently developed empirically and theoretically [e.g. 4–6]. However, it appears the coupling dynamics of drafting were not extended theoretically to more general group dynamics until Olds' analysis of the factors affecting the success or failure of

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separated groups of competitive cyclists [6]. Consequently, Olds' analysis arguably represents the first published analysis of self-organized collective peloton behavior.

We see therefore that along with the growth of complex systems science, research into the collective dynamics of pelotons is in its infancy. Only recently have the collective dynamics of pelotons been recognized to fall within the domain of complex systems science, and thus recognized to exhibit properties that are applicable among a variety of biological and non-biological systems [7–16].

Two main branches of the collective behavior of pelotons are developing. The first branch relates to self-organized pattern formations in non-competitive environments, primarily in the context of their urban traffic patterns [17,18] (''non-competitive pelotons'').

The second major branch relates to self-organized dynamics of pelotons in a competitive environment (''competitive pelotons''), which in turn can be divided into two sub-categories. The first of these sub-categories is an economic, resource based, or game theoretical analysis of cyclists' cooperation and defection strategies, largely derived from the energy savings benefits of drafting [9,10,19,20] (''game theoretical peloton applications''). Somewhat less clearly delineated, but appropriately falling within the sub-category of game theoretical peloton applications is a study of the exogenous influence of league organizational structure on the competitive dynamics of bicycle racing [21].

The second sub-category of research into competitive pelotons relates to physical self-organized formations that emerge from the collective interactions of cyclists in a competitive environment [7,11–16,21–23]. The intersection between principles underlying the collective dynamics of pelotons and energy savings mechanisms of biological systems has been proposed [11], based largely on research identifying energy savings mechanisms in a variety of collectives including bird flocks in flight [24], sperm formations [25,26], huddling penguins [27], fish schools [28], ducklings on-water formations [29], and drafting among dolphins [30]. The intersection of competitive peloton dynamics and other biological collectives may be extended (but not limited to) to include positional analysis of fish based on aerobic capacities [31], similarities of peloton formations and collective formations among coot birds [32], as well as contrary evidence of the metabolic benefits of caterpillar aggregations [33].

2. Mathematical model

A cyclist's power requirement to overcome wind resistance is proportional to the cube of his or her velocity [34]. Power requirements when drafting, for a single rider are reduced by approximately 18% at 32 km/h ($\sim$20 mi/h), 27% at 40 km/h ($\sim$25 mi/h); and by as much as 39% at 40 km/h among a group of eight riders [4]. For two riders, drafting benefit is negligible at speeds below 16 km/h (10 mi/h) [35].

When cycling in groups, cyclists' sustainable speeds increase according to drafting benefits, leaving the sustainable power output unchanged for drafting cyclists. For example, based on power output ranges reported in [36] a drafting cyclist with a hypothetical maximum power output of 349 W can sustain the speed of a stronger rider up to $\sim$52 km/h ($\sim$32 mi/h) on a flat, windless course, and yet may sustain only approximately 41 km/h under the same conditions, without drafting benefit.[1]

Cyclists' maximal sustainable power outputs (''*MSO*'') depend upon individual physiological capacities, and vary as a function of the duration of the output [38]. A cyclist's *MSO* may be determined if her maximal oxygen uptake parameter (VO_{2max}) is known [39]. For the cyclists whose data is applied in this paper, VO_{2max} values are unknown. However, reasonable estimates of these cyclists *MSO*s were derived from publicly available sprint times and corresponding power outputs, as discussed further.

Pelotons frequently divide into smaller groups, as in Fig. 1. Generally, pelotons divide when the power-output reduction benefit of drafting is no longer sufficient to compensate for the differences in strength between weaker and stronger cyclists. For example, as course inclines increase (i.e. hills), drafting benefit diminishes due to reduced speed, while power-output remains high; in such conditions pelotons tend to divide frequently and into numerous groups, as in Fig. 1 (lower left). For flatter terrain pelotons tend to divide less frequently, indicating that drafting benefits are sufficient for weaker cyclists to sustain the speeds of stronger cyclists, as in Fig. 1 (lower right and upper left).

However, even at high speed on relatively flat courses, pelotons may also undergo division due to fatigue induced at sustained high speed or due to coupling instabilities, or a combination of these factors. Coupling due to drafting is inherently unstable as cyclists continuously adjust their positions, periodically exposing following riders to the wind. This necessitates a rapid response from following cyclists in order to maintain optimal drafting position. Following cyclists are particularly susceptible to increased wind exposure on circuitous or narrow courses; cross-winds, or high density configurations when riders compete for optimal drafting positions. High density situations are particularly unstable due to the high probability of crashes at a critical density threshold.

To demonstrate the mechanics of peloton divisions, we further develop the ''peloton-convergence-ratio'' (*PCR*) expression [14]:

$$PCR = \frac{P_{front} - \left(P_{front} * \frac{D}{100}\right)}{MSO}. \tag{1}$$

[1] Approximated by reference to drafting equations in [3,6], and speed to power conversions in [37].