Applied Mathematics and Computation 251 (2015) 132–142

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

# A multi-start variable neighborhood search for solving the single path multicommodity flow problem

H. Masri [a,*], S. Krichen [a], A. Guitouni [b]

[a] LARODEC Laboratory, Institut Supérieur de Gestion de Tunis, University of Tunis, Rue de la liberté, 2000 Bardo, Tunisia
[b] Peter B. Gustavson School of Business, University of Victoria, Victoria, British-Columbia, Canada

ARTICLE INFO

*Keywords:*
Multicommodity flow problem
Routing problem
Variable neighborhood search

ABSTRACT

In this paper, we propose a new global routing algorithm supporting advance reservation. A predefined number of messages are to be routed in a capacitated network including a set of nodes that can be producers and/or consumers of information. We assume that the same information can be held by different sources. We first propose the non-linear mathematical formulation for this problem by extending the single path multicommodity flow formulation. To solve such an $\mathcal{NP}$-hard optimization problem, we develop a multi-start variable neighborhood search method (MVNS). The results of extensive computational experiments across a variety of networks from the literature are reported. For small and medium scale instances, the results are compared with the optimal solution generated by LINGO in terms of time and optimality. For large size instances, a comparison to a state-of-the-art ant colony system approach is performed. The obtained results show that the MVNS algorithm is computationally effective and provides high-quality solutions.



## 1. Introduction

In telecommunication networks, an efficient routing algorithm should find an optimal path for messages transmission so as to satisfy some quality of services (QoS) for end users. Given a limited capacity, an efficient resource reservation mechanism needs to be defined in order to meet these QoS. In general, we can distinguish two types of network resource reservations [17]. The first type is the immediate reservation which results on having dynamic routing algorithms. The second type is, the in-advance reservation, which allows having a routing plan to be followed in later stages. This second type enables to have a global routing algorithm assuming an a priori knowledge of the network structures and the different requests. Such design is prominent in specific applications where voluminous datasets are required to be transferred across the network. We can state as a potential application the grid computing, that introduces new ways of sharing resources across geographically separated sites. Typical computations on grids lead generally to large data transfers. This would overload the network unless advance reservations are made. Another example of application supporting advance reservation might be the routing of video data in a teleconference where different entities are sharing the network resources. It would be reasonable to assume that, with such applications, users should schedule their requests ahead of time so that the routing optimizer can efficiently manage the transmission.

* Corresponding author.
*E-mail addresses:* masri_hela@yahoo.fr (H. Masri), saoussen.krichen@isg.rnu.tn (S. Krichen), adelg@uvic.ca (A. Guitouni).

http://dx.doi.org/10.1016/j.amc.2014.10.123
0096-3003/

Different centralized routing algorithms were designed in the literature assuming advance reservation. In the context of multiple pairs of source–destination nodes, the routing problem is modeled as a multicommodity flow problem (MCFP) [13] where a commodity corresponds to a demand of a telecommunication traffic between two nodes. The basic MCFP model can be described as follows [13]: given a set of commodities and an underlying network structure consisting of a number of nodes and capacitated arcs, the routing algorithm should find an optimal routing policy to transfer the commodities through the network at minimum cost without violating the capacity limits [2]. For a comprehensive survey of MCFPs and the solution approaches, the reader can refer to [1,2,13,16]. Although the complexity of the basic MCFP is polynomial, adding specific constraints to the MCFP turns the problem complexity to be $\mathcal{NP}$-hard. In fact, Holmberg et al. [12] extended the basic MCFP to include side constraints on paths. They explained that in telecommunication applications there are often additional delay or reliability requirements on paths used for routing. These requirements are modeled as side constraints. The problem is solved at optimality using the column-generation method. Furthermore, if only a single path is allowed for the flow of each commodity the problem complexity is $\mathcal{NP}$-hard. This problem is denoted as the unsplittable MCFP or the single path MCFP. Such model is suitable for circuit switched networks, and certain optical networks where a flow cannot be bifurcated. For instance, in optical networks employing wavelength division multiplexing (WDM), to be able to send data from one access node to another, one needs to establish a single route or path, also called a light path, between the source and destination nodes and to allocate a free wavelength on all of the links on the path [6]. The first mathematical formulation of the single path multicommodity problem was introduced by Barnhart et al. [5]. Different solution approaches were developed to solve the single path MCFP. Barnhart et al. [4] proposed an exact method using branch-and-price-and-cut algorithm. However, due to its $\mathcal{NP}$-hardness the problem was generally solved using approximation algorithms [7,3]. More recently, a metaheuristic approach based on ant colony method was developed [6].

In this paper, we propose to study the single path multicommodity flow problem with bandwidth allocation. The problem consists of sending various messages from a set of sources to different destinations through a capacitated network where each arc is characterized by a capacity and a transmission delay. A node in the network can be an information producer (source) or/and an information consumer (destination) or simply a relay node. The same information can be held by different sources. We assume the existence of a centralized routing coordinator managing all information exchange requests and that the data transfer sizes are known beforehand. Since bandwidth is a valuable and scarce resource, an efficient bandwidth management is a key objective of the routing problem. Hence, solving the routing problem consists of generating a single path for each flow with a dedicated bandwidth value along the path. We propose to model this problem as a multi-source single path multicommodity flow problem (MMCF) generalizing the formulation of the single path multicommodity problem [4]. The proposed formulation allows handling the assumption of having multiple sources for each request as well as the bandwidth allocation mechanism.

Given the complexity of the MMCF, we propose to solve it using the multi-start variable neighborhood search (MVNS) method. VNS is a metaheuristic technique [15] which has quickly gained a widespread success as it generates promising results for numerous optimization problems [11]. Its basic idea is a systematic change of neighborhood within an iterative local search. Brimberg [8] proposed a hybrid combination of VNS and random multi-start local search, and they proved the effectiveness of such procedure in various combinatorial problems. We propose to use MVNS in order to solve the MMCF problem. The adopted neighborhood structure consists of changing one or $k$ paths of a current solution. At each iteration, a submodule optimizing the bandwidth allocation is triggered in order to assign a dedicated bandwidth to each path. This submodule solves a uniobjective nonlinear problem with LINGO API. As a local search method, a greedy exploration procedure is proposed. The results of extensive computational experiments across a variety of networks are reported.

The contributions of this paper are twofold:

1. The mathematical formulation of the MMCF extending the single path multicommodity flow problem model by including the multi-source assumption and the bandwidth allocation management.
2. The adaptation and implementation of the VNS metaheuristic. Using different test problems arising from the telecommunications industry [9,6,12], the experimental results show that with the MVNS, promising solutions can be obtained in reasonable CPU time. This fact is proved by performing a comparison with the exact non-linear solver (LINGO) for small sized problems. Furthermore, an empirical comparison with a state-of-the-art ant colony system (ACS) metaheuristic [6] is performed for large instances.

The remainder of the paper is organized as follows. Section 2 describes MMCF problem and states its formulation as a non linear optimization problem. Section 3 presents the MVNS solution approach. Section 4 provides some experimental results demonstrating the efficiency of the MVNS method.

## 2. Problem description

The MMCF problem can be modeled by a directed graph $G = (N, M)$ where $N$ is a set of nodes and $M$ a set of arcs. Each arc $m$ is characterized by a limited capacity $c_m$ that denotes the maximum number of data units which can be transmitted per unit of time, and a lead time $l_m$ that represents the time required to send data through the arc. A set of messages are to be transmitted across the network. A node can be an information provider (source) or/and a consumer requiring an information