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Estimating the parameters of 3-p Weibull distribution through differential evolution

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ABSTRACT

The Weibull distribution is one of the most widely used lifetime distributions in reliability engineering and the estimation of the parameters of this distribution is essential in the most real applications. Maximum likelihood (ML) estimation is a common method, which is usually used to elaborate on the parameter estimation. The working principle of ML estimation method based on maximizing the established likelihood function and maximizing this function formed for the parameter estimation of a three-parameter (3-p) Weibull distribution is a quite challenging problem. In this paper, this problem have been briefly discussed and an effective approach based on the differential evolution (DE) algorithm operators is proposed in order to enhance the estimation accuracy with less system resources. Three explanatory numerical examples are given to show that DE approach which requires significantly less CPU time and exhibits a rapid convergence to the maximum value of the likelihood function in less iterations, provides accurate estimates and is satisfactory for the parameter estimation of the 3-p Weibull distribution.

1. Introduction

A commonly used model in reliability and lifetime studies [1] is the three-parameter Weibull distribution with a probability density function (PDF) given by

$$f_X(x) = \frac{\beta}{\eta}\left(\frac{x-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}}; \quad x > \gamma \geqslant 0,\ \beta > 0,\ \eta > 0 \tag{1}$$

where $\beta > 0, \eta > 0$, and $\gamma \geqslant 0$ are shape, scale, and location parameters, respectively [2].

This distribution was introduced by the Swedish statistician Waloddi Weibull who used it for the first time in 1939 in connection with his studies on the strength of materials [3]. The Weibull distribution has many applications in the areas of engineering [4], biomedical sciences [5], and air quality determination [6]. It is a known fact that three-parameter Weibull distribution family is extremely flexible and can fit very well an extremely wide range of empirical observations [7].

Successful application of Weibull distribution depends on having acceptable statistical estimates of the three parameters. Estimating the parameters of the three-parameter Weibull distribution family is intrinsically a very difficult task. Nosal and Nosal [8] used Monte Carlo methods and array processing language to investigate the performance of the gradient random

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search minimization procedure for fitting a Weibull distribution to a given data set using minimum Kolmogorov–Smirnov distance approach. Bartolucci et al. [9], Bartkute and Sakalauskas [10], Jukic et al. [11], Jukic and Markovich [12], and Markovich and Jukic [13] examined moments method for Weibull distribution. The most common way of estimating the parameters of the density function from the observed data is the maximum likelihood method for Weibull distribution [14–19]. Luus and Jammer [20] and Cousineau [21] reviewed estimation methods for three-parameter Weibull distribution.

Abbasi et al. [15] and Abbasi et al. [17] focused on likelihood method to estimate the parameters of a three-parameter Weibull distribution and employed a simulated annealing (SA) approach to maximize the likelihood function.

The present study focuses on likelihood method, and uses *differential evolution algorithm* to maximize the likelihood function. Differential evolution (DE) algorithm, proposed by Storn and Price [22,23], is a simple but powerful population-based stochastic search technique for solving global optimization problems. It has been used in many real-world applications [24], machine intelligence applications [25], pattern recognition studies [26], and in the area of engineering design [27–35]. Since using DE algorithm to maximize the likelihood function is suitable as well as using the other heuristic methods such as simulated annealing (SA) and genetic algorithm (GA), increase the importance of this paper.

The rest of this paper is organized as follows. Section 2 deals with the notion of maximum likelihood estimation. Section 3 gives some brief information about what the differential evolution algorithm. Section 4 introduces how to use the DE method in the parameter estimation of the 3-p Weibull distribution. DE algorithms with different mutation schemes are investigated in a comparative fashion each other and SA results obtained by Abbasi et al. [15] in Section 5. Section 6 concludes the study.

2. Maximum likelihood estimation of 3-P Weibull distribution

Estimation theory is a cornerstone in statistical, econometric analysis and engineering designs and there has been introduced several techniques to estimate parameters, of which maximum likelihood estimation (MLE), graphical procedure [36], moments method [37,38] and weighted least square method are some of the most interesting ones [39,40]. The MLE method has very desirable properties. It is widely known that MLE estimators are asymptotically unbiased with the minimum variance, and maximum likelihood estimation is a commonly used technique for parameter estimation [41].

Let $x_1, x_2, \ldots, x_n$ be a random sample of size n drawn, at random, from a probability density function, $f_X\left(x; \vec{\theta}\right)$, of unknown parameters, $\vec{\theta}$. The likelihood function is as follows:

$$L = \prod_{i=1}^{n} f_{X_i}\left(x_i; \vec{\theta}\right) \tag{2}$$

where $\vec{\theta}$ is a vector of size m representing the unknown parameters, i.e.

$$\vec{\theta} = (\theta_1, \ldots \theta_m) \tag{3}$$

The aim, here, is to find a vector, say $\vec{\theta}_0$, that maximizes the so-called likelihood function. To maximize L, we may equivalently use its logarithm, say $Ln(L)$.

The maximization is hard to solve when applied to Weibull distribution. The L function for Weibull distribution is as follows:

$$L(x_1, x_2, \ldots, x_n; \gamma, \eta, \beta) = \prod_{i=1}^{n} \frac{\beta}{\eta}\left(\frac{x_i - \gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{x_i-\gamma}{\eta}\right)^{\beta}}; \quad x > \gamma \geqslant 0,\ \beta > 0,\ \eta > 0 \tag{4}$$

where $\vec{\theta} = (\gamma, \eta, \beta)$. Its logarithm will be as follows:

$$Ln(L(x_1, x_2, \ldots, x_n; \gamma, \eta, \beta)) = nLn\left(\frac{\beta}{\eta}\right) + \sum_{i=1}^{n}\left(-\left(\frac{x_i - \gamma}{\eta}\right)^{\beta} + (\beta - 1)Ln\left(\frac{x_i - \gamma}{\eta}\right)\right) \tag{5}$$

After definition of the L (or $Ln(L)$), several numerical methods can be used for maximization of the likelihood objective function. The methods used most often are the derivative-based ones. According to these methods, the maximization (minimization) is performed along a direction that combines gradient vector (vector of first derivatives with respect to parameters) and the Hessian matrix (matrix of second derivatives with respect to parameters) of the objective function. A group of methods named direct search methods perform the maximization (minimization) of the objective function based only on evaluation of the objective function, without the calculation of derivatives.

Although the idea of maximization (minimization) without the calculation of derivatives is appealing, Bard [42] reports that gradient methods outperform direct search methods both in reliability and speed of convergence. Both gradient and direct search methods may be regarded as local search methods, since the search starts from an initial parameter guess and then evolves to a maximum (minimum).

It is very difficult to maximize L (or $Ln(L)$), using ordinary optimization techniques [15,17]. Difficulties are related to the large number of parameters and multimodal nature the objective function. In order to overcome these difficulties, the use of a powerful heuristic method such as differential evolution algorithm may be considered. Differential evolution (DE) has many advantages including simplicity of implementation, is reliable, robust, and in general is considered an effective global optimization algorithm. In this study, DE was used for maximum likelihood estimation.