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Collision and propagation of electromagnetic solitons in an antiferromagnetic spin ladder medium

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ABSTRACT

Soliton collisions constitute one of the central topics of nonlinear-wave dynamics. We demonstrate an inelastic collision between the electromagnetic solitons in a two coupled Heisenberg spin ladder. The interplay of bilinear ferromagnetic coupling with antiferromagnetic rung and diagonal coupling along with the magnetic field component of the electromagnetic wave (EMW) has been studied by solving the Maxwell's equation together with the two coupled Landau–Lifshitz nonlinear spin equations for the magnetization of the medium. The magnetization dynamics of the spin ladder under the influence of EMW is governed by a two coupled generalized derivative nonlinear Schrödinger (CGDNLS) equations. We employ the Bäcklund transformation to solve CGDNLS equations and construct the one and two-soliton solutions. We explicitly construct two soliton solutions to the CGDNLS equations in the framework of Hirota's bilinearization method. We bring out clearly the various features underlying the fascinating shape changing collisions of electromagnetic solitons both in the absence and presence of antiferromagnetic diagonal coupling.

1. Introduction

The study of spin ladder systems continues to receive a great deal of interest from both theoretical and experimental points of view providing new insights into low dimensional quantum systems. Apart from the possible relevance to high-temperature superconductivity [1–3], the interest in spin-ladders was motivated by their rich temperature-magnetic field phase diagram affected by quantum critical fluctuations and the interplay between exchange and quantum fluctuations leads to a host of novel phases much richer than their classical counterparts. By now it is well established that the Heisenberg spin-$\frac{1}{2}$ ladders with an even number of legs are found to have finite energy gap while those with an odd number of legs show gapless spin excitations and these results have been verified experimentally in a number of systems [2]. The interest on spin ladders increased enormously when experimentalist discovered materials like $(VO)_2P_2O_7$ [4] and $Sr_{n-1}Cu_{n+1}O_{2n}$ [5], whose magnetic and electronic structures were analyzed in [6]. Hence from an experimental and theoretical points of view,

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the spin ladders have become a place where to test different ideas concerning strongly correlated systems [7]. In recent years, a general theory of weakly coupled ladders under strong magnetic fields has emerged [8]. Considerable experimental progress in understanding the antiferromagnetic spin ladders was made through the study of the ladder compounds like Piperidinium copper bromide (BPCB) [9], $(CPA)_2CuBr_4$ (CPA = cyclopentylammonium) [10], $Sr_{14}Cu_{24}O_{41}$ [11], $TlCuCl_3$ [12], NaV_2O_5 [13], bis(2,3-dimethylpyridinium) tetrabromocuprate (DIMPY) [14] and IPA – $CuCl_3$ (IPA = isopropyl ammonium) [15]. Spin ladders have applications in diverse fields such as novel superconductors [16], ultracold atoms [17], quantum computing [18] and quark physics [8]. Despite the abundant literature many questions remain as yet unanswered. Under hole-doping these ladders are predicted to pair and may superconduct. Initially, most of the theoretical results concerning ladder systems were obtained from the standard Heisenberg ladder. Several generalized ladders incorporating interactions beyond simple rung and leg exchange have appeared in the literature and demonstrate remarkably interesting behavior [19,20]. Ladder models with ferromagnetic legs and antiferromagnetic rungs with four-spin exchanges along the diagonal have been less spoken, though they exhibit many interesting aspects [21–24]. Recently, the singularity structure of solutions of the coupled Landau–Lifshitz equation representing the spin dynamics of an isotropic antiferromagnetically coupled spin ladder revealed that it is expected to be integrable only when the two ferromagnetic legs are locked together at least at the level of the leading order allowing only small freedom for the individual spins [25]. Also, the nonlinear spin dynamics of a one-dimensional anisotropic spin ladder with ferromagnetic legs and ferromagnetically coupled rungs is governed by a generalized coupled higher order nonlinear Schrödinger equation and the spin excitations are governed by magnetic solitons [26]. More recently, Kavitha et al. demonstrated the sharing of energy among the legs via rung and thus soliton exhibits switching between bistable states leading to the possibility of construction of logic gates [27]. Läuchli, Schmid and Troyer numerically found a new scalar chiral phase in the two leg spin-$\frac{1}{2}$ ladder with four-spin exchange interactions [28]. In recent years, a revival of interest in the study of nonlinear phenomena occurring during the propagation of electromagnetic (EM) waves in magnetic media having a long-range order can be observed. Eventually, the propagation of pulses of electromagnetic radiation in the form of solitons, occupies a special place in such investigations. The electromagnetic field of optical pulses are now used to write and read informations in magnetic disks via interaction between the magnetic field component of the EM field and the magnetization of the disks. Several studies on the propagation of EM wave (EMW) in the ferro/antiferromagnetic medium were carried out in the recent past taking into account the nonlinear nature of the medium [29]. In this case, the magnetic field component of the EM field is found to excite the magnetization of the ferromagnetic medium in the form of solitons and also the small amplitude plane EMW propagates in the form of EM solitons [30]. This type of study also finds applications in magneto-optic recording [31]. In this direction, Daniel et al. demonstrated lossless propagation of many EM signals simultaneously in a charge-free ferromagnetic medium [32]. More recently, Kavitha et al. performed a systematic analysis on the EMW propagation in a weak antiferromagnetic medium and observed that the spin–orbit induced Dzyaloshinskii–Moriya interaction has a profound effect on the nonlinear excitations by the way of admitting breatherlike stable solitary modes [33]. Study of spin ladders with four-spin exchange via diagonal is expected to clarify the possibility of exotic magnetism induced by the EMW propagation along the medium. Recently, four-spin exchange interactions have been of attracting interest in spin ladder models and spin–orbital models, because these interactions in fact appear in many systems [34–37]. Though two leg antiferromagnetic Heisenberg spin ladders show a rung-singlet ground state and excitations [2], it was revealed that four-spin exchanges can induce a gapped staggered dimer phase [38] and a gapless phase [39–41]. Motivated by this, in the present paper we investigate the propagation of EMW in a spin ladder with ferromagnetic legs coupled via antiferromagnetic rungs with a complex diagonal–diagonal exchange interaction. The plan of the paper is organized as follows: In section II, we formulate the physical model for a spin ladder with two ferromagnetic legs with antiferromagnetic rung and diagonal coupling and derive the dynamical equations for both the spin evolution and EMW propagation. In section III, we employ the reductive perturbation method and made a nonuniform expansion of the magnetization and magnetic field along the direction of the propagation of EMW and the magnetization dynamics is governed by a two coupled GCDNLS equations. The GCDNLS equations are solved using Bäcklund transformation in section IV. In section V, we demonstrate the collision scenario of electromagnetic solitons in both the legs of the spin ladder in the framework of Hirota's bilinearization method both in the absence and presence of diagonal coupling parameter. The results are concluded in section VI.

2. Basic propagation equation

We consider the Heisenberg model of an anisotropic spin ladder with two ferromagnetic legs coupled antiferromagnetically. Each ferromagnetic leg (lattice) consists of N spins all pointing in the same direction represented by the spin vectors $\mathbf{S}_{1i}$ and $\mathbf{S}_{2i}, i = 1, 2, \ldots, N$ respectively, provided $\mathbf{S}_{1i}$ and $\mathbf{S}_{2i}$ are arranged in a checker board manner. However, as the spins $\mathbf{S}_{1i}$ and $\mathbf{S}_{2i}$ at the corresponding lattice sites are considered to be connected antiferromagnetically, as they point antiparallel to each other. This is illustrated in Fig. (1). The basic difference between the two sublattice model of an antiferromagnet and the present spin ladder is that unlike the present model, in antiferromagnets the interlattice interaction acts between spins at the adjacent lattice sites [42]. The unit cell comprises four spins and the Hamiltonian is written as

$$H = -\sum_i \left[J_{leg}[\mathbf{S}_{1i} \cdot \mathbf{S}_{1i+1} + \mathbf{S}_{2i} \cdot \mathbf{S}_{2i+1}] - J_{rung}[\mathbf{S}_{1i} \cdot \mathbf{S}_{2i} + \mathbf{S}_{1i+1} \cdot \mathbf{S}_{2i+1}] \right.$$
$$\left. -J_{dia}[\mathbf{S}_{1i} \cdot \mathbf{S}_{2i+1} + \mathbf{S}_{2i} \cdot \mathbf{S}_{1i+1}] - A\left[(S^x_{1i})^2 + (S^x_{2i})^2\right] + \gamma(\mathbf{S}_{1i} + \mathbf{S}_{2i}) \cdot \mathbf{H}\right], \quad |\mathbf{S}_1|^2 = |\mathbf{S}_2|^2 = 1, \tag{1}$$