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A remark on a stochastic logistic model with Lévy jumps

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ABSTRACT

This note is concerned with a famous stochastic logistic equation with Lévy noises. Sufficient and necessary conditions for extinction and permanence are established. The results reveal that the Lévy noise may change the properties of population dynamics significantly. The results also reveal an important property of the Lévy noise: it is unfavorable for the permanence of the population. Some numerical simulations are introduced to validate the analytical results.

1. Introduction

In recent years, stochastic logistic equation driven by Brownian motion has been received great attention (see, e.g., [1–10]). A classical stochastic logistic equation can be expressed as follows

$$dx(t) = x(t)[r - bx(t)]dt + \alpha_1 x(t)dW_1(t) + \alpha_2 x^2(t)dW_2(t), \tag{1}$$

where $b > 0, W_1(t)$ and $W_2(t)$ are standard and independent Wiener processes. [1,2] investigated system (1) with $\alpha_2 = 0$. The authors considered the permanence, extinction and global attractivity of the solutions. [6] studied system (1) with $\alpha_1 = 0$. They obtained the sufficient conditions for permanence and extinction of the solution. System (1) with regime switching was analyzed in [3–5]; Eq. (1) with time delays was developed and discussed in [8].

However, the natural growth of many populations in the real world often suffer sudden environmental perturbations, such as planting, harvesting, epidemics, earthquakes, etc. These phenomena could not be modeled by the classical stochastic system (1). Several authors (see, e.g., [11–18]) have pointed out that we may use the Lévy jump processes to describe these phenomena. Thus in this note, we consider the following logistic equation with jumps

$$dx(t) = x(t^-)\left[(r - bx(t^-))dt + \alpha_1 dW_1(t) + \alpha_2 x(t^-)dW_2(t) + \int_{\mathbb{Y}} \sigma(u)\tilde{\Pi}(dt, du)\right], \tag{2}$$

where $x(t^-)$ is the left limit of $x(t), \tilde{\Pi}(dt, du) = \Pi(dt, du) - \gamma(du)dt, \Pi$ is a Poisson counting measure with characteristic measure γ on a measurable subset $\mathbb{Y}$ of $(0, +\infty)$ with $\gamma(\mathbb{Y}) < +\infty$. The following restriction on Eq. (2) are natural for biological meanings:

$$1 + \sigma(u) > 0, u \in \mathbb{Y}, \tag{3}$$

When $\sigma(u) > 0$, the perturbation stands for increasing of the species (e.g., planting), while $\sigma(u) < 0$ stands for decreasing (e.g., harvesting and epidemics). Under the following assumption,

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(A) There exists a positive constant c such that $\int_{\mathbb{Y}}[\ln(1+\sigma(u))]^2\gamma(du) < c$, we shall show that

Theorem 1. *For system (2),*

(i) If Assumption (A) holds and $\mu < 0$, then $x(t)$ goes to extinction, i.e., $\lim_{t\to+\infty}x(t) = 0$, almost surely (a.s.). Here

$$\mu = r - 0.5\alpha_1^2 - \int_{\mathbb{Y}}(\sigma(u) - \ln(1+\sigma(u)))\gamma(du).$$

(ii) If $\mu > 0$, then $x(t)$ is stochastically permanent, i.e., there are constants $\varsigma_1 > 0, \varsigma_2 > 0$ such that

$$\liminf_{t\to+\infty}\mathcal{P}\{x(t) \geqslant \varsigma_1\} \geqslant 1-\varepsilon, \liminf_{t\to+\infty}\mathcal{P}\{x(t) \leqslant \varsigma_2\} \geqslant 1-\varepsilon.$$

Remark 1. Let Assumption (A) holds, then Theorem 1 establishes the sufficient and necessary conditions for extinction and stochastic permanence of Eq. (2).

Remark 2. Making use of the basic inequality $x - 1 - \ln x \geqslant 0$ for $x > 0$, we can see that $\mu \leqslant r - 0.5\alpha_1^2$. Therefore Theorem 1 reveals an important property of the Lévy noise: it is unfavorable for the permanence of the population. To see this more clearly, consider Eq. (1). [6] have proved that if $r - 0.5\alpha_1^2 > 0$, then the solution of (1) is stochastically permanent. However, Theorem 1 shows that if $\int_{\mathbb{Y}}(\sigma(u) - \ln(1+\sigma(u)))\gamma(du) > r - 0.5\alpha_1^2$, then the solution of model (2) goes to extinction a.s.

Remark 3. [11] have studied the following logistic equation with jumps

$$dx(t) = x(t^-)\left[(r - bx(t^-))dt + \alpha_1 dW_1(t) + \int_{\mathbb{Y}}\sigma(u)\tilde{\Pi}(dt,du)\right]. \quad (4)$$

Under condition (3), the authors have shown that

(a) If Assumption 1 holds and $\mu < 0$, then the solution of Eq. (4) will become extinct a.s.
(b) If $\mu_1 := r - \alpha_1^2 - \int_{\mathbb{Y}}\frac{\sigma^2(u)}{1+\sigma(u)}\gamma(du) > 0$, then the solution of Eq. (4) is stochastically permanent.

Clearly, Eq. (4) is a special case of Eq. (2) ($\alpha_2 = 0$). Thus on the one hand, our Theorem 1 extends the result (a). On the other hand, note that

$$\mu = \mu_1 + 0.5\alpha_1^2 + \int_{\mathbb{Y}}\left[\frac{1}{1+\sigma(u)} - 1 - \ln\frac{1}{1+\sigma(u)}\right]\gamma(du) \geqslant \mu_1.$$

That is to say, our Theorem 1 improves the result (b) greatly.

2. Proof

As a standing hypothesis we assume in this paper that Π, W_1 and W_2 are independent.

Lemma 1. *For any initial value $x_0 > 0$, there is a unique global solution $x(t) > 0$ to model (2) a.s.*

Proof. The proof is similar to that of Theorem 3.1 in [17] by applying Itô's formula to $\sqrt{a} - 1 - 0.5\ln a, a > 0$ and hence is omitted. □

Lemma 2 ([19]). *Suppose that $M(t), \quad t \geqslant 0$, is a local martingale vanishing at time zero. Then*

$$\lim_{t\to+\infty}\Gamma_M(t) < +\infty \quad \Rightarrow \quad \lim_{t\to+\infty}\frac{M(t)}{t} = 0 \quad a.s.,$$

where $\Gamma_M(t) = \int_0^t\frac{d\langle M\rangle(s)}{(1+s)^2}, \quad t \geqslant 0, \quad \langle M\rangle(t) = \langle M,M\rangle(t)$ is Meyer's angle bracket process.

Proof. of (i): Applying Itô's formula to Eq. (3) gives