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A radial basis functions based finite differences method for wave equation with an integral condition

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ABSTRACT

The hyperbolic partial differential equation, which contains integral condition in place of classical boundary condition arises in many application of modern physics and technologies. In this article, we propose a numerical method to solve the hyperbolic equation with nonlocal boundary condition using radial basis function based finite difference method. Several numerical experiments are presented and compared with some existing method to demonstrate the efficiency of the proposed method.

1. Introduction

In recent years the development of numerical methods for the solution of hyperbolic partial differential equations with nonlocal boundary condition have been of great importance in many branches of science and engineering. These nonlocal boundary condition arises when the boundary condition can not be measured directly. Hyperbolic equations in one dimension that contain nonlocal boundary condition have been studied by several authors. In [1], Beilin discussed proof of the existence, uniqueness, and continuous dependence of a solution upon the data, for an initial-boundary value problem that combines Neumann and integral conditions for a hyperbolic equation. In [2], Gordeziani et al. also discussed hyperbolic equations with nonlocal boundary conditions. For more information about the problem, see [3] and reference therein.

In this article, following one dimensional hyperbolic problem is considered which has nonlocal boundary condition in place of standard boundary condition:

$$v_{tt} - v_{xx} = q(x,t), \quad 0 < x < l,\ 0 < t \leqslant T, \tag{1.1}$$

with initial conditions,

$$v(x,0) = f(x), \quad 0 \leqslant x \leqslant l, \tag{1.2}$$
$$v_t(x,0) = g(x), \quad 0 \leqslant x \leqslant l, \tag{1.3}$$

and the Dirichlet boundary condition

$$v(0,t) = m(t), \quad 0 < t \leqslant T, \tag{1.4}$$

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with nonlocal condition

$$\int_0^l v(x,t)dx = E(t), \quad 0 < t \leqslant T, \tag{1.5}$$

where $q, f, g, m(t)$ and $E(t)$ are known functions. We shall assume that q is sufficiently smooth to produce a smooth classical solution v.

In [4], Dehghan presented finite difference schemes to solve the problem. In [3], Saadatmandi et al. proposed a method based on shifted Legendre tau technique for the solution of proposed problem. Ang [5], proposed a numerical scheme based on an integro-differential equation and local interpolating functions for solving the one-dimensional wave equation subject to a non-local conservation condition and suitably prescribed initial-boundary conditions. In [6], a cubic B-spline scaling function together with boundary scaling function are introduced to solve the problem. In [7], Shakeri et al. presented the solution of the one-dimensional nonlocal hyperbolic equation by the method of lines.

A new mesh free method for partial differential equations based on radial basis function is currently undergoing active research. These methods aim to eliminate the structure of the mesh and approximate the solution using a set of random points rather than points from grid discretization.

There has been a considerable interest in the application of radial basis function collocation method to time dependent problems. To cite a few, Zerroukat et al. [8] have used unsymmetric collocation method to solve heat equation, Chinchapatnam et al. [9] solved unsteady convection diffusion equation by unsymmetric and symmetric collocation method. For computing optimal value for the shape parameter of RBF, they used minimal residual error strategy proposed by Cheng et al. [10]. Boztosum et al. [11] present numerical compression of RBF collocation method and finite difference method for numerical solution of 1D and 2D convection diffusion equation. Rocca et al. [12] present the radial basis function hermit collocation approach to solve time dependent convection diffusion equation. Dehghan et al. [13], propose a numerical scheme to solve the one-dimensional hyperbolic equation that combines classical and integral boundary conditions using collocation points and approximating the solution using radial basis functions. Tatari et al. [14] proposed radial basis function based collocation method to solve the one-dimensional parabolic partial differential equation subject to given initial and non-local boundary conditions. It was found that accuracy of RBF based method depend on the shape parameter, choosing optimal value of shape parameter is still an interesting problem for many researchers. Considerable effort has been made in this direction. More details can be found in [15,17,16] and reference therein.

In most papers cited above the main drawback is to invert a highly ill-conditioned dense collocation matrix due to the use of globally supported radial basis function. To resolve shortcoming there are several strategies developed in the literature, few of them as local RBF approach by Lee et al. [18], radial point interpolation method proposed by Liu et al. [19] and RBF based differential quadrature method proposed by Shu et al. [20]. Wright et al. [21] proposed radial basis function finite difference method, the idea is to use radial basis functions with a local collocation as in finite difference mode thereby reducing number of nodes and hence producing a sparse matrix. Kadalbajoo et al. [22–24] extend the localization concept of Wright et al. [21] to solve option price problem. In the present work, a radial basis function based finite difference method to solve the one dimensional wave equation with an integral condition is presented.

The outline of the paper is a follows. In Section 2, we provide a suitable transformation. In Section 3, the complete derivation of radial basis function based finite difference approximation of any operator has been discussed. An implementation of numerical method to the governing problem is presented in Section 4. In Section 5, some results of numerical experiments are given. Finally in Section 6, conclusions are given.

2. Reformulation of the problem

The problem (1.1)–(1.5) describe in the last section is nonstandard because of the unusual boundary condition. The presence of integral conditions makes the application of standard numerical methods much more expensive. So let first reduce the problem (1.1)–(1.5) to an equivalent problem. The first step in this direction is to make a homogeneous boundary condition in Eqs. (1.4) and (1.5). By introducing a new function [13]

$$w(x,t) = v(x,t) - z(x,t), \quad 0 \leqslant x \leqslant l, \; 0 < t \leqslant T, \tag{2.1}$$

where

$$z(x,t) = \left(1 - \frac{2x}{l}\right)m(t) + \frac{2x}{l^2}E(t). \tag{2.2}$$

Now Eqs. (1.1)–(1.5) are converted into the similar equation with Dirichlet condition and integral condition are homogeneous;

$$w_{tt} - w_{xx} = Q(x,t), \tag{2.3}$$

with initial conditions,

$$w(x,0) = \tilde{f}(x), \quad 0 \leqslant x \leqslant l, \tag{2.4}$$

$$w_t(x,0) = \tilde{g}(x), \quad 0 \leqslant x \leqslant l, \tag{2.5}$$