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# An optimum approach of profit analysis on the machine repair system with heterogeneous repairmen

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ABSTRACT

We consider a machine repair problem with queue-dependent heterogeneous repairmen. The stationary probability distribution of the number of failed machines in the system is derived. A profit model is developed to determine the optimal values of threshold to assign/add one more repairman for increasing the total service rate where the service rate is assumed adjustable. The probabilistic global search Lausanne (PGSL) method is employed to find an initial trial solution. Based on this initial solution, a direct search method is applied to obtain the optimal values of the threshold and, then, the Quasi -Newton method is implemented to adjust the corresponding service rates. Some numerical experiments are performed to justify the efficiency of the optimum approach.



## 1. Introduction

In real-life situations, to reduce the situation of congestion and heavy loading, some additional repairmen are prepared to keep the system operational when the machines have failures. In this paper, we consider a machine repair problem with queue-dependent heterogeneous repairmen. For the queueing models with queue-dependent servers, Singh [1] analyzed the infinite capacity M/M/2 and M/M/3 queueing systems with homogeneous and heterogeneous servers. Garg and Singh [2] dealt with the same problem for $M/M/2/\infty$ queueing system and determined the optimal queue length to maximize profit under a cost structure. Yamashiro [3] extended the work of Singh [1] and Garg and Singh [2] to more general cases. Wang and Tai [4] studied a finite capacity M/M/3 queueing system with three heterogeneous servers. Jain [5] investigated a finite capacity M/M/R queueing system with $R$ heterogeneous servers. Lin and Ke [6] used a genetic algorithm to deal with the optimization problem for thresholds of a Markovian queueing system with a triadic policy. Huang et al. [7] considered the controlling arrival and service problem of two-removable-server system using a genetic algorithm. Lin and Ke [8] analyzed an M/M/r queueing system with an infinity capacity in which the number of working servers changes dependent on the queue length.

In the classical machine repair models, it is assumed that the repair rate of the repairmen is independent of queue length. Such machine repair problems have received considerable attentions in the literatures. Ke and Wang [9] studied the multi-server machine repair problem with balking, reneging, and server breakdowns. Haque and Michael [10] surveyed the research published on the machine repair problem in which machines interfere with each other's service. Wang et al.

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[11] considered the single-server machine repair problem with working vacation. Gupta [12] originally analyzed a machine repair problem with warm standbys and server vacations where the single server leaves the system when the repair facility is empty. Gupta's work [12] provided an algorithm to compute the steady-state distribution of the number of failed machines in the system. Gupta's model was extended to cases with server breakdown by Ke [13], who derived system performance measures as well as a cost-sensitivity analysis. Recently, Ke and Wang [14] obtained the steady-state results for a machine repair problem with two types of standby and multi-server vacations. They carried out a sensitivity analysis of the system parameters on the joint optimal parameters. More recently, Ke and Wu [15] studied a multi-server machine repair model with standbys and synchronous multiple vacations.

As mentioned earlier, most existing researches focus on queueing system but not machine repair problems. Yamashiro and Yuasa [16] investigated the M/M/2 and M/M/3 machine repair problems where the number of required repairmen changes depending on the number of failed machines in the system. They derived the steady-state characteristics of the system such as the average number of failed machines and waiting failed machines. It motivates as to investigate a machine repair system with queue-dependent heterogeneous repairmen.

When failed machines in the system increase, the decision-maker often assigns more repairmen to reduce the number of failed machines in the system. A possible application of our model is manufacturing system for printed circuit boards (PCBs). Consider a PCB manufacturing industry which manufactures various products that satisfy various requirements of quality. The process of PCB manufacturing has numerous steps that include manufacturing substrates, drilling and plating holes, and creating the printed circuit pattern on the substrates. The machines may fail randomly. The factory owns multiple machines/robots and the repairman is responsible for repair/maintain the machines when they have failures in a non-preemptive manner. When the number of failed machines/robots becomes larger, more repairmen are needed. On the other hand, the number of repairmen required decreases when failed machines become less. Moreover, various repairmen may have different rates because of different experience and ability. This system can be modeled as a machine repair system with queue dependent heterogeneous repairmen. That is, the number of repairmen employed depends upon the number of failed machines/robots in the system. From the point of view of economic applications, the manager can make decisions with the maximum profit of this system.

This paper has four main objectives: (i) derives the steady-state solutions for the machine repair system with queue-dependent heterogeneous repairmen; (ii) obtains closed-form solutions for some system characteristics; (iii) established a profit function to determine the optimal values including thresholds of the number of failed machines to activate the repairmen and their corresponding repair rates; and (iv) develops a hybrid approach to find the maximum profit. This article is organized as follows: In Section 2, we describe the assumptions of the model. We present the model formulation using a birth-and-death process, and derive the stationary probabilities in Section 3. Then, the steady-state characteristics of the system are presented. In Section 4, by constructing a profit model, we develop a hybrid approach including two heuristic algorithms to obtain the optimal values of threshold and the corresponding service rates. Some numerical examples are showed in Section 5. Finally, conclusions are given in Section 6.

## 2. Model descriptions

We consider a machine repair system with $L = M + W$ identical machines and $R$ queue-dependent heterogeneous repairmen having various mean repair rates. More detailed descriptions about the assumptions of considered queueing system are presented as follows:

1. These $L$ machines are unreliable and may have a breakdown at any time. $M$ of $L$ machines are operating machines and the other $W$ machines are standbys that can replace the failed machines. The failure time of each operating and standby machine is assumed to be independent exponential with mean breakdown rate of $\lambda$ and $\alpha$ $(0 \leqslant \alpha \leqslant \lambda)$, respectively.
2. Failed machines will be sent immediately to the maintenance facility where $R$ heterogeneous repairmen are responsible for repairing the failed machines. For the $j$th repairman, $j = 1, 2, \ldots, R$, the service time is assumed to be exponentially distributed with a mean service rate of $\mu_j$.
3. The failed machines waiting in the facility form a single waiting line and are served in the order of their arrival (FCFS). When a failed machine is repaired, if the number of operating machines in the system is less than $M$ (the system is short), it is sent back to be an operating machine and the failure rate changes from $\alpha$ to $\lambda$. Otherwise, it is held back as a standby machine and keeps the failure rate as $\alpha$.
4. For manpower allocation, the number of repairmen assigned in the system depends on the number of failed machines in the system. The allocation mode follows a threshold policy presented as:
   (1) The first repairman is permanently available in the system.
   (2) For $j = 1, 2, \ldots, R-1$, as soon as there are $N_j$ failed machines in the system, the $(j+1)$th repairman will be assigned for increasing the service speed. Once the number of failed machines in the system is reduced to $N_j - 1$, the $(j+1)$th repairman will be removed from the system and assigned to other jobs.

On the basis of the assumptions described above, the queueing system we investigate can be modeled as an M/M/R machine repair problem with heterogeneous repairmen and a threshold policy. In the next section, the steady-state results are obtained.