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Simulation modeling of a thrown ball bouncing nonlinearly across a grid of cups

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ABSTRACT

This work is motivated by a staple carnival game. A player throws a ping-pong ball onto a grid of cups with the goal of having the ball land in a cup. Though there are many variations to this game, there is a common underlying characteristic. As the ball bounces on the cup grid, its sequence of bouncing trajectories becomes nonlinear. It is this nonlinearity which makes it impossible for an observer to predict the outcome, and makes the game difficult.

The nonlinearity comes from the interaction of the ball's linear motion, angular motion, and the how it bounces off the cup edges. The insight that led to the development of this model is that the ball bouncing on a cup edge is equivalent to it bouncing on a tilted surface. Thus, to develop a predictive model for this game, we modeled a spinning partially elastic ball as it bounces over a series of arbitrarily-tilted surfaces.

We embedded this algorithm in a Monte Carlo simulation model which simulates a player throwing the ball while varying initial launch parameters. Using this model, we were able to track possible trajectories and make probabilistic statements about various outcomes of the game. Furthermore, we used our empirical results to suggest different scenarios for the game, then applied the model to assess and quantify their impacts on difficulty.

Visual inspection and brief analysis of an actual game support our model's credibility.

1. Introduction

Consider a generalization of a common carnival game. A ping-pong ball is thrown by hand, then allowed to bounce along a table until it lands on an adjacent table covered with a grid of cups. *The player's goal is to get the ball to land in a cup*. This setup, as viewed from overhead, is shown in Fig. 1. The bounce table dimensions are (1.0×0.8) m, and the grid of cups is 10×10. We let the table with the grid of cups be slightly lower than the bounce table, so that the cup tops are at the same height as the bounce table surface. The cups themselves are modeled as having a radius of 40 mm; their wall thicknesses are assumed to be negligible, and not doubled where cups meet.

Each time the ball lands on the edge of a cup, the resulting behavior is the same as if the ball had impacted a tilted flat surface. This is because the ball does not usually land on its center bottom, but deflected up the side. The directions of the outgoing motion are changed, based on the angle of impact. Small changes to incoming motion can lead to dramatically altered outgoing motion; the ping-pong ball can bounce straight, bounce back, deflect at a right angle, or anything in between. This nonlinearity is what makes this kind of game challenging, as the bouncing trajectories are impossible for a player to predict.

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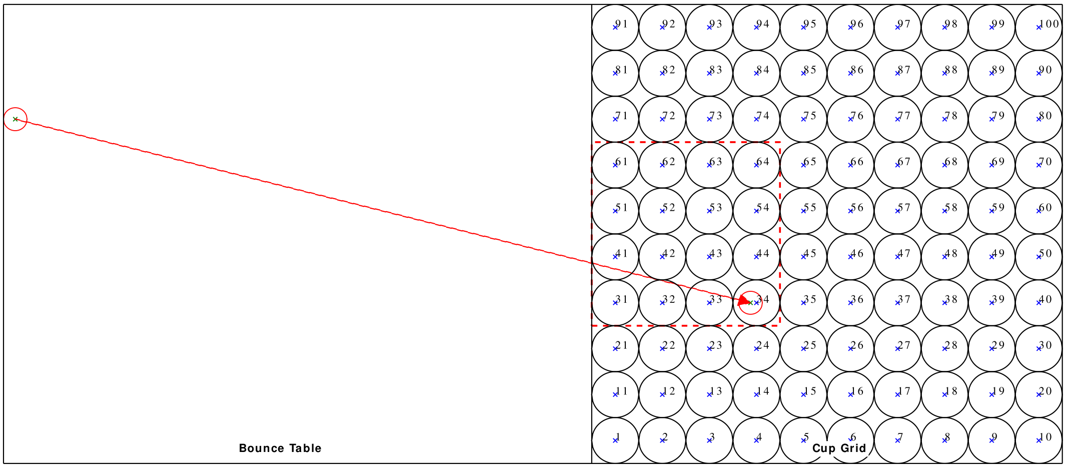

Fig. 1. The ball is thrown from the far left and bounces along a bounce table (1 m length, 0.8 m width) until it reaches an immediately adjacent square table with grid of cups; from here it bounces until it lands in a cup (as shown) or falls of the table.

Generalized, we developed an algorithm for modeling the physical phenomena that control the nonlinear bouncing trajectories of a partially elastic spinning ball over a series of arbitrarily-tilted surfaces. This algorithm is embedded in a Monte Carlo simulation model. Our goal is to simulate a person playing this game, so as to be able to probabilistically evaluate outcomes. Initial launch parameters are accordingly simulated such that the ball will typically first land on the cup grid where we deem a player would aim so as to maximize the chance of success. We call this the *target block*; it is shown as the dashed square in Fig. 1. We apply this model to evaluate modifications to the game and quantify the effects on difficulty.

The physics of balls in motion have been studied for a long time. This article is specifically concerned with the physics behind ball collisions, which has been rigorously studied for at least decades. In the late 1960s, Garwin [1] studied the dynamics of a Superball as it bounced between two parallel surfaces before returning to the thrower. Brody [2] also studied how a ball bounces off a rigid surface at an oblique angle, but using a tennis ball and a different bounce model than Garwin. Relatively simple bounce has been studied for balls from tennis, baseball, basketball, and golf; other subjects have been a Superball, steel ball bearing, plasticene ball, and Silly Putty ball [3,4]. The interactions between linear motion, angular motion, deformation, and friction at the point of impact have been studied by Maw et al. [5] and Cross [6], among others. Cross's study concerned tennis balls and Superballs. The effect of a balls' spin is more of a focus in [7] (tennis) and [8] (ping-pong). Pauchard and Rica [9], Hubbard and Stronge [10], and Cross [11] all studied impact deformation of ping-pong (and other) balls.

Due special mention are Hamilton and Reinschmidt [12] and Okubo and Hubbard [13]. The former because they were the first to perform simulation modeling of the dynamics between a basketball and hoop. In [13], the authors developed a completely general differential equation model for basketball free throw, with distinct sub-models for flight and impacts on the rim, backboard, bridge, and backboard + bridge. They simulated initial launch parameters for the free throw and evaluated the results.

Most of the published literature we reviewed has either addressed the physics of ball collisions from a purely theoretical formulaic perspective, or through frame-by-frame analysis of specialized, controlled collisions. Our contribution to the ball bounce literature is in two areas. Firstly, our article is the first to rigorously model what happens when a spinning elastic ball bounces on a series of surfaces that are arbitrarily-tilted at planes $\pm 90^{\circ}$. Secondly, our study is the first to evaluate the effect of the variation in initial launch parameters in said situation, in the spirit of [12,13].

In Section 2, we detail the physical phenomena controlling the trajectories described by the bouncing ball, and the algorithm for modeling them. In Sections 3 and 4, we describe the simulation study and its results.

2. Newtonian physics of a bouncing ball

According to the International Table Tennis Federation [14], an official ping-pong ball for international competition should have a radius of $r = 20$ mm, mass of $m = 2.7$ g, and normal coefficient of restitution e_z between 0.89 and 0.92. We model the ping-pong ball as a thin-shelled sphere having these quantities for r and m; we use $e_z = 0.90$.

The ball is first hand-thrown from a position at the end of a *bounce table*, as shown in Fig. 2; the seven initial launch parameters shown are: