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# A simple adaptive-feedback scheme for identical synchronizing chaotic systems with uncertain parameters

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ABSTRACT

An adaptive-feedback control scheme for identical synchronizing two linearly coupled chaotic systems and identifying uncertain parameters is proposed. In comparison with the previous methods it inherited, the present scheme is more flexible and accessible. This is because that the prerequisites on the given chaotic systems are much looser, and the adaptive-feedback controllers involved are relatively independent. Moreover, the present scheme is a thoroughly amendatory one by which the uncertain parameters can be identified. Three concrete chaotic systems are employed to show the effectiveness of the presented scheme. In addition, how the coefficient in the adaptive law affects the convergence rate and the final strength is studied numerically.



## 1. Introduction

The intrinsic properties of chaotic systems have a great variety of inherent applications [1]. In this case, the characteristic of chaos should be harnessed to cater to the utilization. In the context of defectiveness of classical control schemes, two pioneering papers opened a new era on the suppression and migration of chaotic motions, namely, O–G–Y method for chaos controlling [2] and P–C method for chaos synchronization [3]. The essence of chaos synchronization is that two chaotic systems follow the same trajectory on the phase space. In the synchronization regime, the drive system drives the response system via the transmitted signals rendering the errors in variables of them are asymptotically stable locally or globally. With the persistent enthusiasm on the study of chaos synchronization, there are a vast variety of synchronization schemes have been proposed and applied in many research fields, such as, active–passive method [4], feedback control method [5], adaptive control method [6], impulse control method [7], and sliding mode method [8]. Among these methods, synchronization of chaotic or hyper-chaotic systems with adaptive control is an effective scheme for adjusting the parameters of controllers and identifying the uncertain parameters of response systems [9–14].

Until now, adaptive feedback control and synchronization of full (or complete) different and identical systems have been studied in depth [15]. In general, nonlinear adaptive feedback control plays a dominant role thereof. Subsequently, plenty of nonlinear adaptive feedback controllers were proposed although most of them are too complex to implement practically. From the viewpoint of practical application, linear feedback control is a simple and unmatched method. Nevertheless, one drawback of linear feedback is the difficulty in finding modest control gains [16], even in the linearizable chaotic systems [17]. In the worst case, some unbounded terms should be involved to achieve synchronization [18]. It may be a universally

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accepted design methodology for linear feedback controls based on sound theoretical issues, but not suitable for engineering implementations at all. Another drawback is that it may waste long time to synchronize all of the states although the chaotic systems are simple in structure [19]. In addition, the control gains are always assigned arbitrarily [20]. To get rid of the difficulty in the try-and-error process, they are usually given as big as possible to ensure or speed up the convergence effectively. In spite of lots of efforts have been paid on the adaptive linear feedback control, it is still a challenging topic to be going on up to now. Based on LaSalle invariance principle of differential equations [21], Huang proposed a simple linear feedback control scheme with adaptive feedback strengths (gains) in his series of papers [22–26]. In the augment system, the feedback coupling terms are linearly relying on the errors in variables. The feedback strengths are updated with the errors in variables dynamically, rather than prescribed arbitrarily in advance. Furthermore, the convergence rate of the adaptive feedback strength is regulated by a factor $\gamma$. Hereby, it can be inferred that the feedback strengths will be very small, while the transient time before arriving at the synchronization regime should be very long, if $\gamma$ is assigned a small number temporally. Usually, the right-hand nonlinear functions of the given systems are assumed to follow uniform Lipschitz condition. This scheme has been adopted extensively by others to realize controls and identical synchronizations [27–37]. Amongst of these extensions, Zheng followed and amended Huangs control scheme [29]. He pointed out that setting variable $x_i = 0$, if $|x_i| \leqslant |x_j|$, therefrom canceling the corresponding interaction term, is not feasible illustrated by the Lorenz system. In addition, he derived his Theorem 1 from a more extreme assumption than the uniform Lipschitz condition [29]. However, the proof on his Theorem 1 may be not right because two terms, $\sum_{i=1}^{n}\delta_{ij}x_i^2$ and $\sum_{i=1}^{n}\delta_{ij}x_i^{2n}$ are not equal. The immediate result is that the transient time is very long, even the coefficient $\gamma$ is very large ($\gamma = 10$) such that the amplitude of adaptive control strength is very big. Based on LaSalle invariance principle of differential systems, Chen, Guo and Vincent made a great progress on choosing the number of the interaction terms [30–36]. The choice of the interaction terms is determined by the largest invariant set contained in $\dot{\boldsymbol{V}}(\boldsymbol{e}(t)) = 0$ for the augment system, rather than canceling the corresponding coupling by setting $\epsilon_{ii} = 0$ if $|e_i| < |e_j|$. As before, a uniform (or local) Lipschitz condition on a given dynamical system should be assumed in advance but never verified. And the choice of coefficient $\gamma$ in the adaptive law of control strength is still arbitrary. The effects on the convergent speed by setting $\gamma$ in a reasonable interval are never considered. In addition, all of their schemes are incapable of tackling the given systems with uncertain parameters. Their researches only concern some special chaotic and hyper-chaotic systems with certain parameters. But, in real-life applications, it is hardly the case that the parameters in coupled chaotic or hyper-chaotic systems can be assumed to be identical all the time. In this case, the practicability of these methods are somewhat limited.

Motivated by these considerations, a new adaptive linear feedback scheme for synchronizing two linearly coupled chaotic systems and identifying their uncertain parameters is proposed. It is not only simple in both analysis and the form of controllers, but is easy to implement in practice. In comparison with the previous methods it inherited [22–37], the present scheme is more flexible and accessible. This is because that the prerequisites on the given chaotic systems are very loose. For example, it only needs that the trajectories along the attractors are bounded; and the linear approximation is the single criterion for stability analysis. The adaptive controllers for identifying the uncertain parameters in response systems are relatively independent of the expression of error dynamical system. The effectiveness of the presented scheme is illustrated by three sets of two linearly coupled chaotic systems. Then we will show how the regulation factor $\gamma$ in the adaptive law affects the convergence rate and the final strength of control gains. Finally, the limitation of the proposed method is also discussed.

## 2. Mathematical formulism

The greatest contribution of the linear feedback control with adaptive strength is that the designing of linear feedback controllers is independent of concrete systems regardless of the number of interaction terms. The variable feedback strength is automatically adapted to completely synchronize almost arbitrary identical chaotic systems. In this sense, the control method is very universal and strict. Moreover, not all of the linear feedback terms are necessary for synchronize all of the states. Which one should be added mandatory to the response system is determined by the largest invariant set contained in $\dot{\boldsymbol{V}} = 0$ for the augment system [22–37]. This is the crucial factor for controlling and synchronizing some special chaotic systems successfully via single controller [31].

Consider a chaotic system, which is given by,

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{F}(\boldsymbol{x}(t), \boldsymbol{\alpha}), \tag{1}$$

where, $\boldsymbol{x}(t) = (x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector. $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_m)^{\mathrm{T}} \in \mathbb{R}^m$ is the known or anticipated parameter vector, which determines the final evolved states of (1) in the parameter space, such as bounded attractors, unbounded repellers, or coexisting form. $\boldsymbol{F}(\boldsymbol{x}(t)) = (f_1(\boldsymbol{x}(t)), f_2(\boldsymbol{x}(t)), \ldots, f_n(\boldsymbol{x}(t)))^{\mathrm{T}} : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is a sufficiently smooth nonlinear function. Without loss of generality, we suppose that $\Omega \in \mathbb{R}^n$ to be a chaotic bounded set in which system (1) is globally attractive. And the vector function $\boldsymbol{f}(\boldsymbol{x}(t))$, is supposed to satisfy the following assumption.

**Assumption 1.** Suppose that $\nabla \boldsymbol{F}(\boldsymbol{x}(t)) = \left(\partial f_i(\boldsymbol{x}(t))/\partial x_j(t)\right)_{1 \leqslant i,j \leqslant n} \subset \Omega$ is bounded at all of the points of every trajectory along the attractor.