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Stabilization of underactuated two-link gymnast robot by using trajectory tracking strategy

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ABSTRACT

This paper concerns the stabilization of an underactuated two-link gymnast robot called acrobot. A trajectory tracking control strategy is presented. First, we carry out a homeomorphous coordinate transformation on the acrobot system that transforms it into a new simplified nonlinear system. And then, a desired motion trajectory is designed for the new system. Finally, we use an equivalent-input-disturbance (EID) method to design a controller that makes the new system asymptotically track the desired trajectory. This enables the acrobot to be swung up from the downward position and to be stabilized at the upright position. The proposed strategy changes the stabilization of the nonlinear acrobot system into that of a linear time-invariant error dynamic system with an artificial disturbance. And it uses a single controller to accomplish the motion control objective of the acrobot. These makes the strategy simple and efficient. Simulation results demonstrate its validity and its superiority over others.

1. Introduction

An underactuated mechanical system (UMS) is characterized by the fact that the number of control inputs is less than that of generalized coordinates. In other words, the number of the degree of freedom (DOF) of an UMS is larger than the number of installed actuators. Many mechanical systems belong to this type, for example, a helicopter, a spacecraft, and an underwater vehicle. Due to the existence of unactuated DOFs, many underactuated systems are not full-state feedback linearizable [1], and some are not even small-time locally controllable [2]. This makes the control of such systems much more difficult than that of fully-actuated ones. And there has been an increasing interest in the problem of controlling an UMS in the past few years [3–5].

A two-link manipulator called acrobot [6] is one of the most studied UMSs by researchers. This manipulator has two DOFs and has only one actuator mounted at the second joint (see Fig. 1). It is a highly simplified model of a human gymnast on a high bar, in which the passive first joint models the gymnast's hands on the bar, and the actuated second joint models the gymnast's hips. In order to mimic the motion of the gymnast going from a natural hanging position to a handstand on the bar, a control objective of swinging up an acrobot from the downward position and balancing it at the upright position has been intensively studied in recent years. And many control strategies have been presented. In [7], Spong firstly used a partial

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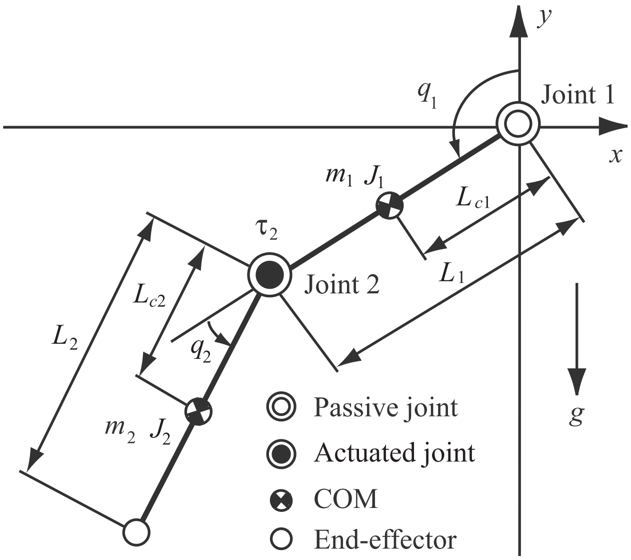

Fig. 1. Model of acrobot.

feedback linearization (PFL) approach to design a swing-up controller that makes the acrobot enter an attractive area around the upright position, and then the controller switches to a LQR-based balancing controller that stabilizes the acrobot at the upright position finally. To employ this kind of switching strategy on the stabilization of the acrobot better, Henmi et al. [8] designed a new reference trajectory for the linear variables of the acrobot appeared in the PFL approach, Xin [9] and Lai et al. [10] used an energy-based to design the swing-up controller, and Lai et al. presented a fuzzy swing-up controller in [11]. All these make the acrobot enter the attractive area more easily. In addition, a robust balancing controller is presented in [12], a Takagi–Sugeno fuzzy balancing controller is presented in [13]. These balancing controller enlarge the basin of the attractive area and make the acrobot be easily captured in this area.

Although the above switching strategy is effective in many cases, it does not guarantee the global stability of the control system. As a result, the balancing controller might not capture the acrobot in the attractive area, and the robot might get out this area. This can cause a long stabilizing time for the acrobot system. Researchers have made a lot of efforts to overcome this shortcoming. On the one hand, some new analytical approaches on the global stability of the switching strategy have been presented, such as a non-smooth Lyapunov function method in [14]. On the other hand, attempts have also been made to use a single controller to achieve the motion control objective of the acrobot. For instance, an integrator backstepping (IB) method, a IDA-PBC method, a quotient method were presented in [15–17], respectively. But the IB method suffers from the problem of the explosion of terms, and the others are only local-control methods. That is, the initial position is required to be in a region around the upright position for both of them. Although a suitable choice of control gains expands the attractive area and may allow the acrobot to be swung up from the downward position, the validity was only illustrated by simulations, but has not been proven theoretically. All of these motivate us to find an effective control method of stabilizing acrobot.

In this paper, we develop a trajectory tracking strategy that enables an acrobot to be swung up from the downward position and to be stabilized at the upright position. The design procedure of the strategy consists of three steps. First, we use a homeomorphous coordinate transformation to transform the acrobot into a new simplified nonlinear system. And then, we design a desired motion trajectory for the new nonlinear system. Finally, we get an error dynamic system about the desired trajectory and use an equivalent-input-disturbance (EID) method to design a controller that drives the error system to converge to the origin. As a result, the stabilization of the acrobot at the target point is achieved along a desired motion trajectory. The proposed strategy does not need to divide the motion space of the acrobot, and just uses a single controller to accomplish system's motion control objective. Moreover, it changes the stabilization of a complicated nonlinear system (i.e., acrobot) into that of a time-invariant error dynamic system with an artificial disturbance. It is simple and efficient. Most importantly, the stabilizing movement and settling time of the acrobot can be accurately predicted from the desired trajectory.

2. Motion equations and coordinate transformation

The model of an acrobot is shown in Fig. 1, in which q_1 is the angle of the first link relative to the vertical, q_2 is the angle of the second link relative to the first link, m_i is the mass of the i–th link $(i = 1, 2), L_i$ is the length of the i–th link $(i = 1, 2), L_{ci}$ is the distance from the i-th joint to the center of mass (COM) of the i-th link $(i = 1, 2), J_i$ is the moment of inertia around the COM of the i-th link $(i = 1, 2)$, τ_2 is the control input torque applied on the second joint, and g is the acceleration of gravity.