Applied Mathematics and Computation 253 (2015) 294–307

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

An analysis of a family of Maheshwari-based optimal eighth order methods

Changbum Chun [a], Beny Neta [b,*]

[a] Department of Mathematics, Sungkyunkwan University, Suwon 440-746, Republic of Korea
[b] Naval Postgraduate School, Department of Applied Mathematics, Monterey, CA 93943, USA

ARTICLE INFO

Keywords:
Iterative methods
Order of convergence
Basin of attraction
Extraneous fixed points
Weight functions

ABSTRACT

In this paper we analyze an optimal eighth-order family of methods based on Maheshwari's fourth order method. This family of methods uses a weight function. We analyze the family using the information on the extraneous fixed points. Two measures of closeness of an extraneous points set to the imaginary axis are considered and applied to the members of the family to find its best performer. The results are compared to a modified version of Wang–Liu method.

Published by Elsevier Inc.

1. Introduction

"Calculating zeros of a scalar function f ranks among the most significant problems in the theory and practice not only of applied mathematics, but also of many branches of engineering sciences, physics, computer science, finance, to mention only some fields" [1]. For example, to minimize a function $F(x)$ one has to find the points where the derivative vanishes, i.e. $F'(x) = 0$. There are many algorithms for the solution of nonlinear equations, see e.g. Traub [2], Neta [3] and the recent book by Petković et al. [1]. The methods can be classified as one step and multistep. One step methods are of the form

$$x_{n+1} = \phi(x_n).$$

The iteration function ϕ depends on the method used. For example, Newton's method is given by

$$x_{n+1} = \phi(x_n) = x_n - \frac{f(x_n)}{f'(x_n)}. \tag{1}$$

Some one point methods allow the use of one or more previously found points, in such a case we have a one step method with memory. For example, the secant method uses one previous point and is given by

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n).$$

In order to increase the order of a one step method, one requires higher derivatives. For example, Halley's method is of third order and uses second derivatives [4]. In many cases the function is not smooth enough or the higher derivatives are too complicated. Another way to increase the order is by using multistep. The recent book by Petković et al. [1] is dedicated to multistep methods. A trivial example of a multistep method is a combination of two Newton steps, i.e.

* Corresponding author.
E-mail addresses: cbchun@skku.edu (C. Chun), bneta@nps.edu (B. Neta).

http://dx.doi.org/10.1016/j.amc.2014.12.064
0096-3003/Published by Elsevier Inc.

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},$$
$$x_{n+1} = y_n - \frac{f(y_n)}{f'(y_n)}. \quad (2)$$

Of course this is too expensive. The cost of a method is defined by the number (ℓ) of function-evaluations per step. The method (2) requires four function-evaluations (including derivatives). The efficiency of a method is defined by

$$I = p^{1/\ell},$$

where p is the order of the method. Clearly one strives to find the most efficient methods. To this end, Kung and Traub [5] introduced the idea of optimality. A method using ℓ evaluations is optimal if the order is $2^{\ell-1}$. They have also developed optimal multistep methods of increasing order. See also Neta [6]. Newton's method (1) is optimal of order 2. King [7] has developed an optimal fourth order family of methods depending on a parameter β

$$w_n = x_n - \frac{f(x_n)}{f'(x_n)},$$
$$x_{n+1} = w_n - \frac{f(w_n)}{f'(x_n)}\left[\frac{1+\beta r_n}{1+(\beta-2)r_n}\right], \quad (3)$$

where

$$r_n = \frac{f(w_n)}{f(x_n)}. \quad (4)$$

Maheshwari [8] has developed the following optimal fourth order method

$$w_n = x_n - \frac{f(x_n)}{f'(x_n)},$$
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\left[r_n^2 - \frac{1}{1-r_n}\right], \quad (5)$$

Table 1
The eight cases for experimentation.

Case	Method	g	a
1	LQ	–	0.7
2	LQ	-	2.1
3	QQ	0.8	0.6
4	QQ	1.8	2
5	QC	−0.3	0.6
6	QC	−3.6	2
7	LQ	–	2
8	WLN	–	–

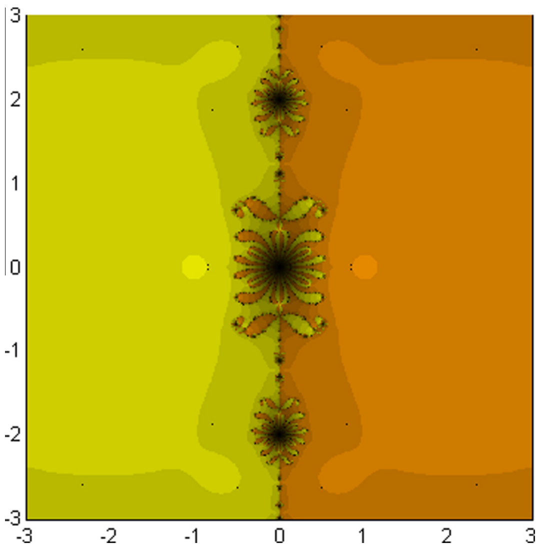

Fig. 1. LQ case 1 for the roots of the polynomial $z^2 - 1$.