Applied Mathematics and Computation 253 (2015) 324–344

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Double orbit finite retrial queues with priority customers and service interruptions

Madhu Jain [a], Amita Bhagat [a,*], Chandra Shekhar [b]

[a] Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee 247 667, India
[b] Department of Mathematics, Birla Institute of Technology and Science, Pilani, Rajasthan 333 031, India

ARTICLE INFO

Keywords:
Double orbits
Unreliable server
Finite capacity
Threshold recovery policy
Cost function
Cellular radio network

ABSTRACT

The present study deals with the double orbit finite capacity retrial queues with unreliable server. The system facilitates the arrival of two types of customers known as priority and non priority customers and can hold a maximum of L priority customers and K non-priority customers as per its capacity. The priority customers are served prior to the non-priority customers. Moreover, the server is unreliable which may breakdown while servicing either priority or non-priority customer. The failed server is sent for repair following threshold recovery policy to become as good as earlier. Both transient as well as steady state analysis of the model has been done using by matrix method. Various performance measures including queue length, reliability metrics, long run probabilities, etc. have been obtained using various state probabilities. The application of the model to cellular radio network has been discussed. The cost function has been constructed and optimized using meta heuristic approach. The sensitivity analysis of various performance indices has been performed as an illustration.

1. Introduction

The formation of queues in front of railway reservation counters; doctor's clinic, shopping malls etc. are day to day scenes which we encounter everyday in almost all the spheres of life throughout the world. Sometimes, due to congestion situations customers are deprived of service and hence decide to retry later on as soon as the congestion reduces or server becomes free. This state of reattempts for service gives rise to special queues known as retrial queues. These queues are the result of the fact that a customer who finds the server unavailable for service joins orbit which is a virtual pool of customers from where he tries again for the service as soon as he finds the server free. Enormous work has been done by a number of researchers in the direction of retrial queues and the related literature can be found in the survey articles by Yang and Templeton [1], Artalejo [2,3] Artalejo and Falin [4] and Artalejo [5]. The detailed account on retrial queues are given in the books on retrial queues by Falin and Templeton [6] and Artalejo and Coral [7].

In real life situations, the server seems to be unreliable due to which the service of the customers may be interrupted before its completion. It is worthwhile to have a look into the important works done on retrial queues. Li and Zhao [8] investigated queueing model with impatient customers in addition to constant retrial rate and unreliable server. An M/M/1 retrial queue with unreliable server has been investigated by Sherman and Kharoufeh [9] with infinite orbit capacity. Yang and Alfa

* Corresponding author.
E-mail addresses: madhufma@iitr.ernet.in (M. Jain), bhagatamita03@gmail.com (A. Bhagat), chandrashekhar@bits-pilani.ac.in (C. Shekhar).

http://dx.doi.org/10.1016/j.amc.2014.12.066
0096-3003/

[10] studied a class of multi server queueing systems with server failures. Wang and Yang [11] used Quasi-Newton method to study the M/M/1/K queueing system with F-policy and unreliable server. Jain et al. [12] studied an unreliable M/M/2/K queueing system under (*N*,*F*) policy with multi optional phase repair.

To assign priority to one type of jobs over other type of jobs is now a feature of daily routine activities. Now-a-days, while analyzing the queueing scenarios, almost every area seems to be affected by the priority service of one customer over another customer. Choi and Chang [13] analyzed various single server retrial queues with priority calls and many more features like geometric loss, feedback, etc. Artalejo et al. [14] made stationary analysis of a retrial queue with preemptive repeated attempts. Dimitriou [15] investigated a mixed priority retrial queue with negative arrivals, unreliable server and multiple vacations and explored stability conditions as well as the system state probabilities of the system.

It usually happens in machining environment that the repairman is called upon only when a certain minimum number of faults are present in the system and are to be repaired as it is economical both in terms of time and expenses. Efrosinin and Semenova [16] studied M/M/1 system with unreliable server and threshold based repair policy. Efrosinin and Winkler [17] examined a Markovian retrial queueing system with non-reliable server. Jain and Bhagat [18] investigated a finite population retrial queueing system with threshold recovery and unreliable server with geometric arrivals and impatient customers. Purohit et al. [19] analyzed finite queue with threshold recovery and state dependent arrival rates. Jain and Bhagat [20] also studied a finite capacity retrial queueing system with finite capacity and impatient customers.

The transient behavior of various queueing systems has also been studied by several researchers. Anisimov [21] studied averaging methods for the transient behavior in retrial queueing systems. Krinik et al. [22] examined the transient probabilities of single server retrial system using randomization solution. A new formula was developed for the transient solution of Erlang queueing system by Leonenko [23].

The present problem is framed in a very novel manner in the sense that the concept of double orbits is incorporated along with unreliable server and priority customers. No such work has been done as per authors' information. The problem is equipped with retrial orbits having fixed capacity for both the priority and non priority customers. Moreover, the broken down server is repaired following threshold recovery for both priority as well as non priority customers. Non priority customers are served only if no priority customers are present in the system. The transient state solution of the model has been explored using matrix method. To highlight the practical importance of the model under consideration, numerical simulation has been done using an illustration. The cost function has been optimized to determine the optimal parameters. The rest of the paper is organized in the following manner. Section 2 deals with the description of the model with various assumptions and notations. The governing equations are established in Section 3. Various performance measures have been obtained in Section 4. An application on cellular mobile network has been developed in Section 5 and its numerical simulation has been done in Section 6. Section 7 deals with cost optimization of the system. Finally, the conclusions are drawn in Section 8.

2. Model description

We consider an unreliable single server finite retrial queueing model with two types of customers; priority and non-priority customers with double orbits. The basic operation of the model can be described as:

(i) Arrival and retrial process: Two class of customers namely priority and non-priority customers arrive in the system. The priority (non-priority) customers follow Poisson process with mean arrival rate λ_1 (λ_2). On finding the server idle or unavailable for the service, the customers on their arrival may either join the queue in the front of server or they can wait for their turn in their respective retrial orbits. The queue in the front of server can hold a maximum of L priority and K non-priority customers. Further, we assume that the capacity of retrial orbit 1(orbit 2) is of $L-1$ ($K-1$) priority (non-priority) customers.

The priority (non priority) customers retry for their service from their respective orbits; the retrial time is exponentially distributed with rate θ_1 (θ_2). The retrial process occurs only when single types of customers (either priority or non-priority) are present in the system. However, if both types of customers are present in the system then the priority customers are served like a classical queue and no retrial phenomenon takes place in such a case.

(ii) Service process: All the customers are served following the discipline of first come first serve (FCFS); the priority and non priority customers are served according to exponential distribution with rates μ_1 and μ_2, respectively. The priority customers are served prior to the non-priority customers. The server after serving the last priority customer present in the queue automatically starts the servicing of non-priority customers waiting for the service. There is no retrial mechanism in this case.

(iii) Breakdown state: The server is unreliable and may break down while serving the customers. The server failures occur in Poisson fashion with rates α_1 (α_2) while servicing priority (non priority) customers. The server failures occur while he is busy in serving either type of customer. No break down occurs during retrial and idle states. In case when the server breakdowns occurs, the service of the customer already in service is resumed and continued as soon as the repair process of the server is completed. Moreover, the system transition from repair states is allowed only into the busy states and not in the retrial states.