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Spiral periodic structures in a parameter plane of an ecological model

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ABSTRACT

We investigate a parameter plane of a set of three autonomous, ten-parameter, first-order nonlinear ordinary differential equations, which models a tri-trophic food web system. By using Lyapunov exponents, bifurcation diagrams, and trajectories in the phase-space, to numerically characterize the dynamics of the model in a parameter plane, we show that it presents typical periodic structures embedded in a chaotic region, forming a spiral structure that coils up around a focal point while period-adding bifurcations take place.

1. Introduction

Investigations in parameter-planes of continuous-time nonlinear dynamical systems, using Lyapunov exponents to numerically characterize the dynamics by discriminating periodic, quasiperiodic, and chaotic behaviors, have grown substantially in recent years [1–47]. Note that we have cited above only papers published since 2010. Typical self-organized periodic structures have been observed embedded in chaotic regions of these parameter planes. Some of these are the shrimp-shaped periodic structures, first described by Gallas [48] in a discrete-time nonlinear dynamical system, and after observed also in a wide range of both discrete- and continuous-time nonlinear dynamical systems, modeled respectively by maps and sets of first-order ordinary differential equations. Many properties of these periodic structures have been described since then, showing that they may appear highly organized in different ways in a parameter plane of a particular system. This is true for a large number of different systems, in different fields, including chemical reactions, population dynamics, electronic circuits, lasers, forced oscillators, neural networks, among others. One such type of organization consists of a set of shrimp-shaped periodic structures forming a spiral that coils up around a focal point while period-adding bifurcations take place. To our knowledge these spiral bifurcations have been observed in parameter planes of electronic circuits [1,29,49], a Rössler model [50], a chemical oscillator [1], a Hopfield neural network [30], modified optical injection semiconductor lasers [31], and a tumor growth mathematical model [43]. The spiral periodic structures were experimentally detected in electronic circuits [51], and the global mechanism responsible for its origin and organization was reported simultaneously by Vitolo et al. [14] and Barrio et al. [15].

Here we report the period-adding spiral bifurcation of the shrimp-shaped periodic structures in a parameter plane of a tri-trophic food web system [52], which is modeled by a set of three autonomous, ten parameter, first-order ordinary differential equations. The Letter is organized as follows. In Section 2 the mathematical model of the considered tri-trophic food web system is established. In Section 3 parameter planes and other numerical results are presented and interpreted. The Letter is finalized in Section 3.

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2. The model

The equations of motion that theoretically describe the dynamical behavior of the tri-trophic food web system here considered, were recently proposed by Priyadarshi and Gakkhar [52], and are given by

$$\begin{aligned}\dot{X} &= a_0X\left(1-\frac{X}{K}\right)-\frac{a_1XY}{1+b_1X}-\frac{a_2XZ}{1+b_2X+b_3Y},\\ \dot{Y} &= Y\left(-r+\frac{a_1cX}{1+b_1X}\right)-\frac{a_3YZ}{1+b_2X+b_3Y},\\ \dot{Z} &= S_0\left(Z^2-\frac{Z^2}{S_3+S_1X+S_2Y}\right),\end{aligned} \tag{1}$$

where X is the bottom prey population density, Y is the specialist predator population density, and Z is the generalist predator population density. The parameter a_0 is the intrinsic growth rate of the bottom prey, K is the carrying capacity of the environment, a_1 is the maximum grazing rate of the specialist predator Y with respect to bottom prey X, b_1 is the coefficient of food taken by the specialist predator Y with respect to bottom prey X, a_2 is the maximum grazing rate of the generalist predator Z with respect to bottom prey X, a_3 is the maximum grazing rate of the generalist predator Z with respect to the specialist predator Y, and b_2 and b_3 represent the coefficients of food taken by the generalist predator, respectively from X and Y. Parameters r and c are respectively the death rate of the specialist predator in absence of the bottom prey, and its conversion efficiency, S_0 is the intrinsic growth rate of the generalist predator, S_1 and S_2 are the respective food preference of the generalist predator, and S_3 relates to the residual reduction in the generalist predator Z, because of the severe scarcity of the foods X and Y. For details on the role played by each component of the dynamical system, as well as the meaning of each of the fourteen parameters, see Ref. [52] and references therein.

In order to reduce the number of parameters, adequate dimensionless variables and parameters can be introduced, as defined in Ref. [52], to obtain

$$\begin{aligned}\dot{x} &= x\left(1-x-\frac{y}{1+w_1x}-\frac{z}{1+w_2x+w_3y}\right),\\ \dot{y} &= y\left(-w_5+\frac{w_4x}{1+w_1x}-\frac{w_6z}{1+w_2x+w_3y}\right),\\ \dot{z} &= w_7z^2-\frac{w_8z^2}{1+w_9x+w_{10}y},\end{aligned} \tag{2}$$

where all parameters $w_i, i = 1,10$ are positive. All the results derived in the present investigation, which are reported in the next section, were obtained from the non-dimensional form Eqs. (2).

3. Numerical results

Fig. 1 shows a (w_3, w_2) parameter plane for the tri-trophic food web nondimensionalized model (2), which is a 2-dimensional cross-section of their 10-dimensional parameter-space, obtained by plotting the largest Lyapunov exponent (LLE) on a $10^3 \times 10^3$ mesh of points. Following Ref. [52], the remaining parameters in Eqs. (2) were kept fixed as $w_1 = 1.4$, $w_4 = 1.0, w_5 = 0.16, w_6 = 0.1, w_7 = 0.1, w_8 = 0.5, w_9 = 8.0$, and $w_{10} = 8.0$. System (2) was integrated using a fourth order Runge–Kutta algorithm with a fixed time step equal to 10^{-3}, and considering 10^6 steps to evaluate the average involved

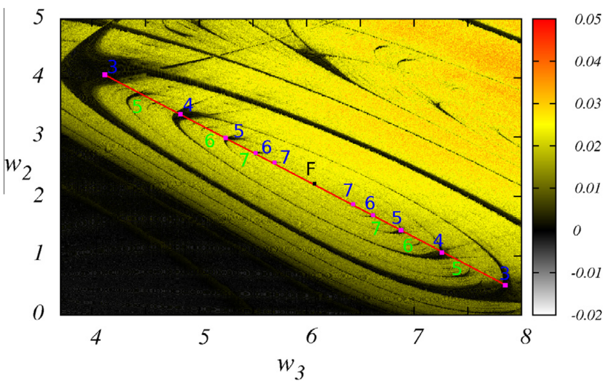

Fig. 1. The (w_3, w_2) parameter plane for system (2) showing two spirals. Color is related to the magnitude of the largest Lyapunov exponent. Number is related to the period of the respective structure (see text). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)