Applied Mathematics and Computation 254 (2015) 20–29

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Controllability of fractional integro-differential evolution equations with nonlocal conditions

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ARTICLE INFO

Keywords:
Nonlocal Cauchy problem
Fractional integro-differential evolution equation
Controllability
Measure of noncompactness
Fixed point theorem

ABSTRACT

This paper concerns the controllability for a class of fractional integro-differential evolution equations with nonlocal initial conditions. By using the fractional calculus, measure of noncompactness and the Mönch fixed point theorem, we obtain a controllability result for the nonlocal Cauchy problem of the fractional integro-differential evolution equations involving noncompact semigroups and the nonlocal functions without Lipschitz continuity. An example is given to illustrate the effectiveness of the abstract results.

1. Introduction

In the past two decades, there were a lot of research works on the fractional differential equations, which have been proved to be valuable tools in the modeling of many phenomena in engineering, physics and economics, etc. The basic properties of various fractional evolution equations have also been investigated. We refer the reader to [1,9,10,12,13,16,19–22,28,29,31,32,35] and the reference therein.

As remarked by the authors of [2,6–8,11,17,23–27,36] and the reference therein, the nonlocal Cauchy problems have better effects in applications than the traditional Cauchy problem with ordinary initial datum. For example, Byszewski [7] obtained the existence and uniqueness of solutions to a class of abstract functional differential equations with nonlocal conditions of the form

$$\begin{cases} x'(t) = f(t, x(t), x(a(t))), \quad t \in J := [0, b], \\ x(0) + \sum_{k=1}^{m} c_k x(t_k) = x_0, \end{cases} \tag{1.1}$$

where $b > 0$ is a constant, $0 < t_1 < t_2 < \cdots < t_m < b, f : J \times X \times X \to X$ and $a : J \to J$ are given functions, X is a Banach space, $x_0 \in X, c_k \in \mathbb{R}, c_k \neq 0 (k = 1, 2, \ldots, m), m \in \mathbb{N}$. It pointed out that if $c_k \neq 0 (k = 1, 2, \ldots, m)$, then the results of [7] can be applied to kinematics to determine the location evolution $t \mapsto x(t)$ of a physical object for which we do not know the positions $x(t_1), \ldots, x(t_m)$, but we know that the nonlocal condition in (1.1) holds. In applications, the nonlocal condition in (1.1) is common, for example

$$\sum_{k=1}^{m} c_k x(t_k) = \sum_{k=1}^{m} \frac{1}{3k} x(t_k). \tag{1.2}$$

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http://dx.doi.org/10.1016/j.amc.2014.12.145
0096-3003/

We can choose $c_k = \frac{1}{3k}(k = 1,2,\ldots,m)$. It is clear that $c_k \neq 0(k = 1,2,\ldots,m)$. Moreover, the nonlocal condition in (1.1) has also been used by Deng [11] to describe the diffusion phenomenon of small amount of gas in a transparent tube.

Controllability is one of the fundamental concepts in mathematical control theory, which plays an important role in control systems. In recent years, the controllability problems for various linear and nonlinear deterministic and stochastic dynamic systems have been studied in many publications using different approaches (cf., e.g., [3–5,13,15,20,28,29,31,34,35] and the reference therein). There have also been some papers (e.g, [15,17]) about the study of the exact controllability of systems represented by nonlinear evolution equations in infinite dimensional spaces. But when the semigroup is compact and other hypotheses are satisfied, the application of exact controllability results is just restricted to the finite dimensional space (cf. [15]). Therefore, a nature problem is: *how to investigate the exact controllability of the fractional evolution equations involving noncompact semigroups?*

In this paper, we investigate the exact controllability of the following nonlocal Cauchy problem for the fractional integro-differential evolution equations in Banach spaces X involving noncompact semigroups and the nonlocal functions without Lipschitz continuity

$$\begin{cases} D^q x(t) + Ax(t) = f(t, x(t), Gx(t)) + Bu(t), & t \in J, \\ x(0) = \sum_{k=1}^{m} c_k x(t_k), \end{cases} \tag{1.3}$$

where D^q denotes the Caputo fractional derivative of order $q \in (0,\ 1), -A : D(A) \subset X \to X$ is the infinitesimal generator of a C_0-semigroup $T(t)(t \geqslant 0)$ of uniformly bounded linear operator, the control function u is given in $L^2(J, U), U$ is a Banach space, B is a linear bounded operator from U to X, f is a given function will be specified later and

$$Gx(t) = \int_0^t K(t,s)x(s)ds$$

is a Volterra integral operator with integral kernel $K \in C(\Delta, \mathbb{R}^+), \Delta = \{(t,s) : 0 \leqslant s \leqslant t \leqslant b\}$. Throughout this paper, we always assume that

$$K^* = \sup_{t \in J} \int_0^t K(t,s)ds.$$

In this work, by using a special type of nonlocal function, we delete the compactness and Lipschitz continuity of nonlocal function, only assume that $c_k(k = 1,2,\ldots,m)$ satisfy the condition $(H0)$ (see in Section 2). We firstly introduce a suitable definition of mild solutions of the system (1.3), and then we prove the exact controllability of the system (1.3) by using the Mönch fixed point theorem under the case of noncompact semigroup.

The rest of this paper is organized as follows. In Section 2, some preliminaries are given regarding to fractional calculus and measure of noncompactness. In Section 3, we prove the exact controllability of the nonlocal fractional evolution system (1.3). An example is given in Section 4 to illustrate the effectiveness of the abstract results.

2. Preliminaries

Let X be a Banach space with norm $\|\cdot\|$. We denote by $C(J,X)$ the Banach space of all continuous X-value functions on interval J with the norm $\|u\|_C = \max_{t \in J} \|u(t)\|$. We use $\|f\|_{L^p}$ to denote the $L^p(J, \mathbb{R}^+)$ norm of f whenever $f \in L^p(J, \mathbb{R}^+)$ for some p with $1 \leqslant p < \infty$. Let $A : D(A) \subset X \to X$ be a closed linear operator and $-A$ generate a C_0-semigroup $T(t)(t \geqslant 0)$ of uniformly bounded linear operator in X. That is, there exists a constant $M \geqslant 1$ such that $\|T(t)\| \leqslant M$ for all $t \geqslant 0$.

Let us recall the following well known definitions and facts in fractional calculus. For more details, see [1,13,16,9–22,29,35] and the reference therein.

Definition 1. The Caputo fractional derivative of order $n - 1 < \sigma < n$ with the lower limits zero for a function $f \in C^n[0, \infty)$ can be written as

$$D^\sigma f(t) = \frac{1}{\Gamma(n-\sigma)} \int_0^t (t-s)^{n-\sigma-1} f^{(n)}(s)ds, \quad t > 0,\ n \in \mathbb{N}.$$

For $x \in X$, we define two families $\{S(t)\}_{t \geqslant 0}$ and $\{V(t)\}_{t \geqslant 0}$ of operators by

$$S(t)x = \int_0^\infty \eta_q(\theta) T(t^q\theta)xd\theta, \quad V(t)x = q\int_0^\infty \theta\eta_q(\theta)T(t^q\theta)xd\theta, \quad 0 < q < 1,$$

where

$$\eta_q(\theta) = \frac{1}{q}\theta^{-1-\frac{1}{q}}\rho_q(\theta^{-\frac{1}{q}}), \quad \rho_q(\theta) = \frac{1}{\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\theta^{-qn-1}\frac{\Gamma(nq+1)}{n!}\sin(n\pi q), \quad \theta \in (0, \infty).$$