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Flow and heat transfer over a moving surface with non-linear velocity and variable thickness in a nanofluids in the presence of Brownian motion

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ABSTRACT

The effects of variable thickness, hydromagnetic flow, Brownian motion, heat generation, on heat transfer characteristics and mechanical properties of a moving surface embedded into cooling medium consists of water with nano-particles are studied. The governing boundary layer equations are transformed to ordinary differential equations. These equations are solved analytically using (OHAM) for general conditions. The velocity, temperature, and concentration profiles within the boundary layer are plotted and discussed in details for various values of the different parameters such as Brownian parameter, thermophoresis parameter, shape parameter, magnetic parameter and heat source parameter the effect of the cooling medium and flatness on the mechanical properties of the surface are investigated.

1. Introduction

The problem of boundary layer flow over a moving surface into a cooling medium is a mathematical simulation to the heat treatment process. The process of heat-treating is the method by which metals heated and cooled in a series of specific operations that never allow the metal to reach the molten state. The purpose of heat-treating is to make a metal more useful by changing or restoring its mechanical properties. Through heat-treating, we can make a metal harder, stronger, and more resistant to impact. Heat-treating can also make a metal softer and more ductile.

The boundary layer flow caused by a moving surface has drawn the attention of many researches [1–17]. The dynamics of the boundary layer flow over a moving surface embedded into a regular fluid was the main goal of many researchers until the advent of new type of fluid, which called a nanofluid.

Nanofluid described as a fluid in which solid nanoparticles with the length scales of 1–100 nm suspended in conventional heat transfer basic fluid. These nanoparticles enhance thermal conductivity and convective heat transfer coefficient of the base fluid significantly. Conventional heat transfer fluids such as oil, water and ethylene glycol mixture are poor heat transfer fluids because the thermal conductivity affects the heat transfer coefficient between the heat transfer medium and the heat transfer surface. Therefore, numerous methods have been taken to improve the thermal conductivity of these fluids by suspending nano/micro sized particle materials in liquids. The term nanofluid has been suggested by Choi [18]. There are many

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studies on the mechanism behind the enhanced heat transfer characteristics using nanofluids. The collection of papers on this topic is included in the book by Das et al. [19] and in the review papers by Azizah et al. [20], Aminreza et al. [21], Nazar et al. [22], Hamad [23], Oztop et al. [24], Yacob et al. [25], prasad [26], and Elbashbeshy et al. [27].

On the other hand, the effect of Brownian motion and thermophoresis of a nanofluid have been investigated by Rana [28], Alsaedi [29], Khan [30], and Anbuchezhian [31].

All of the previous studies deal with moving surface with constant thickness under different effects of flow and thermo boundary layer. The variable thickness may occur in the engineering applications more frequently than a flat surface. Fang et al. [32] studied the boundary layer flow over a stretching sheet with variable thickness. Elbashbeshy et al. [33] studied the flow and heat transfer over a moving surface with non-linear velocity and variable thickness in a nanofluid.

The objective of the present paper is to study the effect of hydro-magnetic flow and heat transfer characteristic of a nanofluid over a steady moving surface with variable thickness on the mechanical properties of the surface in the presence of Brownian motion and heat source during the heat-treating process.

2. Formulation of the problem

Consider a steady, laminar, two dimensional flow of an incompressible viscous electrically conduction nanofluid over a continuous moving surface in the presence of a transverse magnetic field $B(x)$ and heat generation $Q(x)$. We assume that the surface is sufficiently thin with no induced stream-wise pressure gradients and the induced magnetic field produced by the motion of an electrically conducting fluid is negligible. This assumption is valid for small magnetic Reynolds number. Further, since there is no external electric field, the electric field due to polarization of charges is negligible. Moreover, it is assumed that both the fluid phase and nanoparticles are in thermal equilibrium state and no slip occurs between them.

Fig. 1 shows the x-axis runs along the center of the surface, and the y-axis is perpendicular to it.

We assume that the surface is not flat with a given profile, which is specified as $y = \delta(x+b)^{\frac{1-n}{2}}$, we assume the coefficient δ being small so that the surface is sufficiently thin.

The governing boundary layer equations for the steady two-dimensional laminar hydro-magnetic nanofluid flow over a moving surface and subjected heat source can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{Q(x)}{\rho Cp}(T - T_\infty) \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

With boundary conditions

$$\begin{aligned} &u = U_w, \quad v = 0, \quad T = T_w, \quad C = C_w, \quad \text{at } y = \delta(x+b)^{\frac{1-n}{2}} \\ &u = 0, \quad v = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{as } y \to \infty \end{aligned} \tag{5}$$

where u and v are velocity components in the x and y directions, respectively, υ is the kinematic viscosity, ρ is the density of the base fluid, σ is the electrical conductivity, and α is the thermal diffusion, D_B is the Brownian diffusion coefficient, D_T is the Thermophoretic diffusion coefficient, τ is the ratio between the effective heat capacity of the nanoparticle and heat capacity of the fluid, $B(x)$ is the strength of the magnetic field. The special form of the magnetic field $B(x) = B_0(x+b)^{\frac{n-1}{2}}$ and heat

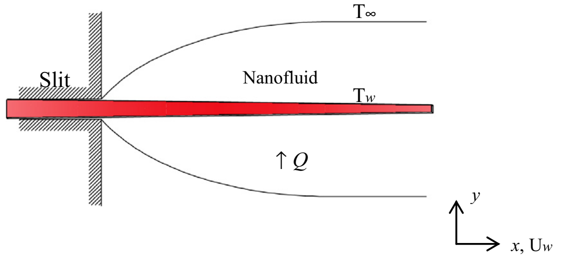

Fig. 1. Physical model and coordinate system.