Applied Mathematics and Computation 254 (2015) 252–265

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Existence, uniqueness and almost surely asymptotic estimations of the solutions to neutral stochastic functional differential equations driven by pure jumps

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ARTICLE INFO

Keywords:
Neutral stochastic functional differential equations
Pure jumps
Existence and uniqueness
Exponential estimations
Almost surely asymptotic estimations

ABSTRACT

In this paper, we are concerned with neutral stochastic functional differential equations driven by pure jumps (NSFDEwPJs). We prove the existence and uniqueness of the solution to NSFDEwPJs whose coefficients satisfying the local Lipschitz condition. In addition, we establish the p-th exponential estimations and almost surely asymptotic estimations of the solution for NSFDEwJs.

1. Introduction

Stochastic delay differential equations (SDDEs) have come to play an important role in many branches of science and industry. Such models have been used with great success in a variety of application areas, including biology, epidemiology, mechanics, economics and finance. In the past few decades, qualitative theory of SDDEs have been studied intensively by many scholars. Here, we refer to Mohammed [1], Mao [2–5,9], Buckwar [6], Kuchler [7], Hu [8], Xu [10], Wu [11], Appleby [12], Gyongy [13] and references therein. Recently, motivated by the theory of aeroelasticity, a class of neutral stochastic equations has also received a great deal of attention and much work has been done on neutral stochastic equations. For example, conditions of the existence and stability of the analytical solution are given in [14–20]. Various efficient computational methods are obtained and their convergence and stability have been studied in [21–25].

However, all equations of the above mentioned works are driven by white noise perturbations with continuous initial data and white noise perturbations are not always appropriate to interpret real data in a reasonable way. In real phenomena, the state of neutral stochastic delay equations may be perturbed by abrupt pulses or extreme events. A more natural mathematical framework for these phenomena has been taken into account other than purely Brownian perturbations. In particular, we incorporate the Levy perturbations with jumps into neutral stochastic delay equations to model abrupt changes.

In this paper, we study the following neutral stochastic functional differential equations with pure jumps (NSFDEwPJs)

$$d[x(t) - D(x_t)] = f(x_t, t)dt + \int_U h(x_t, u)N_{\tilde{p}}(dt, du), \quad t_0 \leqslant t \leqslant T. \tag{1}$$

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http://dx.doi.org/10.1016/j.amc.2014.12.126

To the best of our knowledge, there are no literatures concerned with the existence and asymptotic estimations of the solution to NSFDEwPJs (1). On the one hand, we prove that Eq. (1) has a unique solution in the sense of L^P norm. We do not use the fixed point Theorem. Instead, we get the solution of Eq. (1) via successive approximations. On the other hand, we study the p-th exponential estimations and almost surely asymptotic estimations of the solution to Eq. (1). By using the It$\widehat{o}$ formula, Taylor formula and the Burkholder Davis inequality, we have that the p-th moment of the solution will grow at most exponentially with exponent M and show that the exponential estimations implies almost surely asymptotic estimations. Although the way of analysis follows the ideas in [2], however, those results on the existence and uniqueness of the solution in [2] cannot be extended to the jumps case naturally. Unlike the Brown process whose almost all sample paths are continuous, the Poisson random measure $N_{\bar{p}}(dt,du)$ is a jump process and has the sample paths which are right-continuous and have left limits. Therefore, there is a great difference between the stochastic integral with respect to the Brown process and the one with respect to the Poisson random measure. It should be pointed out that the proof for NSFDEwPJs is certainly not a straightforward generalization of that for NSFDEs without jumps and some new techniques are developed to cope with the difficulties due to the Poisson random measures.

The rest of the paper is organized as follows. In Section 2, we introduce some notations and hypotheses concerning Eq. (1). In Section 3, the existence and uniqueness of the solution to Eq. (1) are investigated. In Section 4, we prove the p-th moment of the solution will grow at most exponentially with exponent M and show that the exponential estimations implies the almost surely asymptotic estimations.

2. Preliminaries

Let $(\Omega,\mathcal{F},P)$ be a complete probability space equipped with some filtration $(\mathcal{F}_t)_{t\geqslant t_0}$ satisfying the usual conditions (i.e. it is right continuous and $(\mathcal{F}_{t_0})$ contains all P-null sets). Let $\tau>0$, and $D([-\tau,0];R^n)$ denote the family of all right-continuous functions with left-hand limits φ from $[-\tau,0]\to R^n$. The space $D([-\tau,0];R^n)$ is assumed to be equipped with the norm $||\varphi||=\sup_{-\tau\leqslant t\leqslant 0}|\varphi(t)|$ and $|x|=\sqrt{x^\top x}$ for any $x\in R^n$. If A is a vector or matrix, its trace norm is denoted by $|A|=\sqrt{\text{trace}(A^\top A)}$, while its operator norm is denoted by $||A||=\sup\{|Ax|:|x|=1\}$. $D^b_{\mathcal{F}_0}([-\tau,0];R^n)$ denotes the family of all almost surely bounded, $\mathcal{F}_0$-measurable, $D([-\tau,0];R^n)$ valued random variable $\xi=\{\xi(\theta):-\tau\leqslant\theta\leqslant 0\}$. Let $t_0\geq 0, p\geqslant 2, \mathcal{L}^p_{\mathcal{F}_{t_0}}([-\tau,0];R^n)$ denote the family of all $\mathcal{F}_{t_0}$ measurable, $D([-\tau,0];R^n)$-valued random variables $\varphi=\{\varphi(\theta):-\tau\leqslant\theta\leqslant 0\}$ such that $E\sup_{-\tau\leqslant\theta\leqslant 0}|\varphi(\theta)|^p<\infty$.

Let $(U,\mathcal{B}(U))$ be a measurable space and $\pi(du)$ a σ- finite measure on it. Let $\{\bar{p}=\bar{p}(t),t\geqslant t_0\}$ be a stationary $\mathcal{F}_t$-Poisson point process on U with a characteristic measure π. Then, for $A\in\mathcal{B}(U-\{0\})$, here $0\in$ the closure of A, the Poisson counting measure $N_{\bar{p}}$ is defined by

$$N_{\bar{p}}((t_0,t]\times A):=\sharp\{t_0<s\leqslant t,\bar{p}(s)\in A\}=\sum_{t_0<s\leqslant t}I_A(\bar{p}(s)),$$

where $\sharp$ denotes the cardinality of a set. For simplicity, we denote: $N_{\bar{p}}(t,A):=N_{\bar{p}}((t_0,t]\times A)$. It follows from [26] that there exists a σ- finite measure π satisfying

$$E[N_{\bar{p}}(t,A)]=\pi(A)t,\quad P(N_{\bar{p}}(t,A)=n)=\frac{e(-t\pi(A))(\pi(A)t)^n}{n!}.$$

This measure π is called the Levy measure. Then, the measure $\widetilde{N}_{\bar{p}}$ is defined by

$$\widetilde{N}_{\bar{p}}([t_0,t],A):=N_{\bar{p}}([t_0,t],A)-t\pi(A),\quad t>t_0.$$

We refer to Ikeda [26] for the details on Poisson point process.

The integral version of Eq. (1) is given by the equation

$$x(t)-D(x_t)=x_{t_0}-D(x_{t_0})+\int_{t_0}^t f(x_s,s)ds+\int_{t_0}^t\int_U h(x_s,u)N_{\bar{p}}(ds,du),\tag{2}$$

where

$$x_t=\{x(t+\theta):-\tau\leqslant\theta\leqslant 0\}$$

is regarded as a $D([-\tau,0];R^n)$-valued stochastic process. $f:D([-\tau,0];R^n)\times[t_0,T]\to R^n$ and $h:D([-\tau,0];R^n)\times U\to R^n$ are both Borel-measurable functions. The initial condition x_{t_0} is defined by

$$x_{t_0}=\xi=\{\xi(t):-\tau\leqslant t\leqslant 0\}\in\mathcal{L}^p_{\mathcal{F}_{t_0}}([-\tau,0];R^n),$$

that is, ξ is an $\mathcal{F}_{t_0}$-measurable $D([-\tau,0];R^n)$-valued random variable and $E||\xi||^p<\infty$. $\widetilde{N}_{\bar{p}}(dt,du)$ is the compensated Poisson random measure given by

$$\widetilde{N}_{\bar{p}}(dt,du)=N_{\bar{p}}(dt,du)-\pi(du)dt,$$

here $\pi(du)$ is the Levy measure associated to $N_{\bar{p}}$.