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Controllability of impulsive matrix Lyapunov systems

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ABSTRACT

In this paper, we establish some sufficient conditions for the complete controllability of linear and semilinear impulsive matrix Lyapunov systems. For the semilinear systems, we assume that nonlinearities are Lipschitz type or monotone type. Few illustrative examples are given to compare and substantiate the results of the paper.

1. Introduction

Many of the real life systems arising in science and engineering are control theoretic in nature. Often, the main aim is to compel the given system to behave in a desired manner. This can be achieved mainly by analyzing the controllability of the system. The study of controllability plays a pivotal role in computation of steering controls, controllable decomposition, pole placements and optimal control problems. Vast literature is available on the controllability of linear systems, for instance, refer to, [1–4]. Moreover, controllability of nonlinearly perturbed linear systems, called semilinear systems, has also been established in the literature, see [5,6] and references their in.

In recent years, there has been growing interests towards the study of controllability of impulsive systems, that is, the systems in which the system-state is subject to sudden change or impulse at discrete time points. Study of such systems has received much attention in the literature due to the fact that many evolutionary processes, for instance, some motions of missiles or aircraft, control models in economics, frequency modulated systems and bursting rhythm models in biology, are impulsive in nature (see [7,8]). Controllability of impulsive dynamical systems has been well investigated in the literature, for example, [6,9–13]. Furthermore, controllability of impulsive systems with delay in control is studied by several authors, for example, refer to [14,15] and references their in. Recently, in [16,17] controllability and observability of matrix time-varying impulsive systems are investigated.

In this article, we investigate complete controllability of the following matrix Lyapunov systems with impulse effects

$$\begin{cases} \dot{X}(t) = A(t)X(t) + X(t)B(t) + F(t)U(t) + G(t, X(t)), & t \neq t_k,\ t \in [t_0, T], \\ X(t_k^+) = [I_n + D^k U(t_k)]X(t_k), & k = 1, 2, \ldots, \rho, \\ X(t_0) = X_0, \end{cases} \tag{1.1}$$

where the state $X(t)$ is an $n \times n$ real matrix, control $U(t)$ is an $m \times n$ real matrix. $A(t), B(t), F(t)$ are $n \times n, n \times n, n \times m$ real matrices with piecewise continuous entries and $t_0 \leqslant t_1 \leqslant t_2 \ldots \leqslant t_\rho \leqslant T$ are the time points at which impulse control $U(t_k)$ is given to the system. For each $k = 1, 2, \ldots, \rho, D^k U(t_k)$ is an $n \times n$ diagonal matrix such that $D^k U(t_k) = \sum_{i=1}^m \sum_{j=1}^n d_{ij}^k U_{ij}(t_k) I_n$, where I_n is the identity matrix on $\mathbb{R}^n$ and $d_{ij}^k \in \mathbb{R}$. $G(\cdot,\cdot) : \mathbb{R}^+ \times \mathbb{R}^{n\times n} \to \mathbb{R}^{n\times n}$ is a nonlinear

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function and satisfies the "Caratheodory conditions", that is, $G(\cdot, x)$ is measurable with respect to t for all $x \in \mathbb{R}^{n\times n}$ and $G(t,\cdot)$ is continuous with respect to x for almost all $t \in [t_0, T]$. The control $U(t)$ is said to be impulsive if at $t = t_k, k = 1, 2, \ldots, \rho$, the pulses are regulated and chosen arbitrarily in rest of the domain.

Indeed, the controllability of many special cases of system (1.1) has been studied in the literature. For example, if $B(t) = 0$ and $G(t, x) = 0$ hold along with $D^kU(t_k) = 0$ for $k = 1, 2, \ldots, \rho$, then the system (1.1) reduces to linear time-varying control system whose controllability is well established in the literature, for example, [1,3,2]. Leela et al. [10] studied the controllability of a special case of (1.1), that is, when $B(t) = 0, G(t, x) = 0$, and $A(t), F(t)$ are constant matrices. In [6], complete controllability of a special case of system (1.1) with $B(t) = 0$ is investigated. Murty et al. in [18] studied the controllability of linear non-impulsive matrix Lypunov systems, that is, system (1.1) with $G = 0$ and $D^kU(t_k) = 0$ for $k = 1, 2, \ldots, \rho$. Furthermore, in [19] controllability of semilinear non-impulsive matrix Lyapunov systems, that is, system (1.1) with $D^kU(t_k) = 0$ for $k = 1, 2, \ldots, \rho$, is established. The results of this paper extend and generalize some of the results in [19,18,10,6].

In this paper, first, we obtain sufficient conditions for the complete controllability of unperturbed (linear) matrix Lyapunov system, that is, system (1.1) with $G = 0$. We then establish complete controllability of perturbed (nonlinear) system, that is, system (1.1) itself. The remainder of the paper is organized as follows:

In Section 2, we state some of the basic definitions and properties related to Kronecker products. In Section 3, complete controllability of unperturbed system, that is, system (1.1) with $G = 0$, is established. In Section 4, sufficient conditions for the complete controllability of perturbed system (1.1) are obtained. Finally, we conclude the paper with few illustrative examples provided in Section 5.

2. Preliminaries

Throughout the paper $\mathbb{R}$ denotes the set of all real numbers. $\mathbb{R}^+$ denotes the set of all non-negative real numbers. $\mathbb{R}^{m\times n}$ and $\mathbb{C}^{m\times n}$ denote the set of all $m \times n$ real matrices and $m \times n$ complex matrices, respectively. I_n denotes the $n \times n$ identity matrix. Given any matrix A, $\sum A$ denotes the sum of the absolute values of entries of A. $\langle \cdot, \cdot \rangle$ denotes the standard inner product in $\mathbb{R}^n$ unless otherwise stated.

We start with some basic definitions related to Kronecker products which we shall use in this paper.

Definition 2.1 [20]. Let $A \in \mathbb{C}^{m\times n}$ and $B \in \mathbb{C}^{p\times q}$ then the Kronecker product of A and B is written as $A \otimes B$ and is defined to be the partitioned matrix

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B \end{bmatrix}$$

which is an $mp \times nq$ matrix and in $\mathbb{C}^{mp\times nq}$.

Definition 2.2. Let $A = [a_{ij}] \in \mathbb{C}^{m\times n}$. We denote

$$\widehat{A} = \mathrm{Vec}(A) = \begin{bmatrix} A_{.1} \\ A_{.2} \\ \vdots \\ A_{.n} \end{bmatrix}_{mn\times 1}, \quad \textit{where} \quad A_{.j} = \begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{bmatrix}, (1 \leqslant j \leqslant n).$$

The Kronecker product satisfies the following properties [20]:.

1. $(A \otimes B)^T = (A^T \otimes B^T)$.
2. $(A \otimes B)^{-1} = (A^{-1} \otimes B^{-1})$.
3. $(A \otimes B)(C \otimes D) = (AC \otimes BD)$, provided the dimensions of various matrices are compatible with matrix product.
4. If $A(t)$ and $B(t)$ are matrices, then
$$\frac{d}{dt}(A(t) \otimes B(t)) = \frac{d}{dt}(A(t)) \otimes B(t) + A(t) \otimes \frac{d}{dt}(B(t)).$$
5. $\mathrm{Vec}(AYB) = (B^T \otimes A)\mathrm{Vec}(Y)$.
6. If A and X are matrices of order $n \times n$, then
   (i) $\mathrm{Vec}(AX) = (I_n \otimes A)\mathrm{Vec}(X)$.
   (ii) $\mathrm{Vec}(XA) = (A^T \otimes I_n)\mathrm{Vec}(X)$.

We will now state some essential definitions and results from nonlinear functional analysis that we shall use in this paper. The following contraction principle will be used in this chapter.