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Thermoelastic interaction in a thermally conducting cubic crystal subjected to ramp-type heating

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ABSTRACT

In this paper, the thermoelastic interactions in a homogeneous, thermally conducting cubic crystal, elastic half-plane has been studied. A linear temperature ramping function is used to more realistically model. The general solution obtained is applied to a specific problem of a half space subjected to ramp-type heating. The components of displacement, stresses, and temperature distribution are obtained by applying a numerical finite element method. Some particular cases are also discussed in the context of the problem. The comparison in Lord and Shulman (LS), Green and Lindsay (GL) and Green and Naghdi (GN) theories have been shown graphically to estimate the effect of ramping parameter of heating for isothermal boundary.

1. Introduction

The generalized theories of thermoelasticity have been developed to overcome the infinite propagation speed of thermal signal predicted by classical theory of thermoelasticity Biot [1]. There are two generalizations of the classical theory of thermoelasticity. The first generalization was proposed by Lord and Shulman [2] and is known as L–S theory, which involve one relaxation time for a thermoelastic process. The second generalization is due to Green and Lindsay [3] and is known as G–L theory that takes into account two parameters in relaxation time. Dhaliwal and Sherief [4] extended the generalized theory of thermoelasticity (LS) to anisotropic media. Banerjee and Pao [5] investigated the propagation of plane harmonic waves in homogenous anisotropic thermoelastic solids. Sharma and Singh [6] investigated the propagation of generalized thermoelastic waves in cubic crystals. Li [7] developed the generalized theory of thermoelasticity for an anisotropic medium.

Green and Naghdi [8–10] proposed three models, which are subsequently referred to as GN-I, II, III models. The linearized version of model-I correspond to classical thermoelastic model. In model-II the internal rate of production of entropy is taken to be identically zero implying no dissipation of thermal energy. This model admits undamped thermoelastic waves in a thermoelastic material and is known as thermoelasticity without energy dissipation. Model –III includes the previous two models as special cases and admits dissipation of energy.

Tzou [11] proposed a dual phase-lag heat conduction model to incorporate the effect of microscopic interactions in the fast transient process of heat transport mechanism in a macroscopic formulation. Two different phase–lags (one for the heat

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flux vector and the other for the temperature gradient) have been introduced in the constitutive relation between heat flux vector and the temperature gradient. Thermoelasticity theory corresponding to dual-phase-lag heat conduction was proposed by Chandrasekharaiah [12].

Youssef [13] discussed problem of generalized thermoelastic infinite medium with cylindrical cavity subjected to a ramp-type heating. Youssef and El-Bary [14] used the state space approach to study the problem of thermoelastic infinite layer subjected to ramp-type thermal and mechanical loads. Ezzat and Youssef [15] studied the problem of state space approach for conducting magneto-thermoelastic medium with variable electrical and thermal conductivity subjected to ramp-type heating.

The finite element method is a powerful technique originally developed for numerical solution of complex problem in structural mechanics, and it remains the method of choice for complex system. A further benefit of this method is that it allows physical effects to be visualized and quantified regardless of experimental limitations.

Tay [16] studied the effect of thermoelastic coupling in the determination of transient temperature fields in composite layer by using finite element analysis. Yi and Matin [17] developed a finite element formulation for solving the problem related to thermoelastic damping in beam resonator systems. Abbas and Youssef [18] proposed a general finite element model to analyze transient phenomena in two temperature thermoelastic solids. Othman and Abbas [19] used the finite element method to study the effect of rotation on plane waves at the free surface of a fiber-reinforced thermoelastic half-space. Zad et al. [20] studied the behavior of thermoelastic waves at the interface of a layered medium by using finite element method. Kumar et al. [21] investigated plane deformation due to thermal source in fraction order thermoelastic media. Sheikholeslami et al. [22] studied the effects of heat transfer in flow of nanofluids.

The present investigation is to determine the components of displacements, stresses and temperature distribution in a homogenous, thermally conducting cubic crystal, elastic half-space due to ramp-type heating. Problem investigated here has practical utility in the field of engineering, fiber–wound composites and laminated composite materials.

2. Formulation of the problem

We consider a homogenous, thermally conducting cubic crystal, elastic half-space in the undeformed state at uniform temperature T_0. The rectangular Cartesian coordinate system (x, y, z) having origin on the plane surface $x = 0$ with x-axis pointing vertically into medium is introduced. For two-dimensional problem (xz-plane), all the quantities depend only on space coordinates x, z and time t. The boundary of the half-space is assumed to be affected by ramp-type heating. We take, the displacement vector $\vec{u} = (u, 0, w)$ and temperature $T(x, z, t)$, then the field equations and constitutive relations for such a medium can be written, by following the equations given in a general form (see Green and Lindsay [3] and Dhaliwal and Sherief [4]) as

$$c_{11}\frac{\partial^{2u}}{\partial x^2} + c_{44}\frac{\partial^2 u}{\partial z^2} + (c_{12} + c_{44})\frac{\partial^2 w}{\partial x \partial z} - \beta\frac{\partial}{\partial x}\left(T + \tau_1\frac{\partial T}{\partial t}\right) = \rho\frac{\partial^2 u}{\partial t^2}, \tag{1}$$

$$c_{44}\frac{\partial^2 w}{\partial x^2} + c_{11}\frac{\partial^2 w}{\partial z^2} + (c_{12} + c_{44})\frac{\partial^2 u}{\partial x \partial z} - \beta\frac{\partial}{\partial z}\left(T + \tau_1\frac{\partial T}{\partial t}\right) = \rho\frac{\partial^2 w}{\partial t^2}, \tag{2}$$

$$Kn^*(T_{,xx} + T_{,zz}) = \rho c_e\left(n_1 + \tau_0\frac{\partial}{\partial t}\right)\dot{T} + T_0\beta\left(n_1\frac{\partial}{\partial t} + n_0\tau_0\frac{\partial}{\partial t}\right)(u_{,x} + w_{,z}), \tag{3}$$

and

$$t_{xx} = c_{11}\frac{\partial u}{\partial x} + c_{12}\frac{\partial w}{\partial z} - \beta\left(T + \tau_1\frac{\partial T}{\partial t}\right), \quad t_{zx} = c_{44}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right), \tag{4}$$

$$t_{zz} = c_{12}\frac{\partial u}{\partial x} + c_{11}\frac{\partial w}{\partial z} - \beta\left(T + \tau_1\frac{\partial T}{\partial t}\right), \tag{5}$$

where $\beta = (c_{11} + 2c_{12})\alpha$, here c_{ij} are the isothermal elastic parameters, ρ is the density, c_e is the specific heat at constant strain and τ_0, τ_1 are the thermal relaxation times, α the coefficient of thermal expansion, K is the coefficient of thermal conductivity, u and w are the displacement components along x and z directions respectively, t is time, T is temperature, t_{zx} and t_{zz} are stress components, The thermal relaxation times τ_0 and $\tau_1$satisfy the inequality $\tau_1 \geqslant \tau_0 \geqslant 0$ for the G–L theory only. The field equations of thermally conducting cubic crystal for three different generalizations take the form as

(i) Lord–Shulman (L–S) theory, $\tau_1 = 0$, $\tau_0 > 0$, $n^* = n_0 = n_1 = 1$, the Eqs. (1)–(3) take the form as

$$c_{11}u_{,xx} + c_{44}u_{,zz} + (c_{12} + c_{44})w_{,xz} - \beta T_{,x} = \rho\ddot{u}, \tag{6}$$

$$c_{44}w_{,xx} + c_{11}w_{,zz} + (c_{12} + c_{44})u_{,xz} - \beta T_{,z} = \rho\ddot{w} \tag{7}$$

$$K(T_{,xx} + T_{,zz}) = \rho c_e\left(1 + \tau_o\frac{\partial}{\partial t}\right)\dot{T} + T_o\beta\left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)(u_{,x} + w_{,z}) \tag{8}$$