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Stability, Hopf bifurcations and spatial patterns in a delayed diffusive predator–prey model with herd behavior

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ABSTRACT

In this paper, we consider a delayed diffusive predator–prey model with herd behavior. Firstly, by choosing the appropriate bifurcation parameter, the stability of the positive equilibria and the existence of Hopf bifurcations, induced by diffusion and delay respectively, are investigated by analyzing the corresponding characteristic equation. Then, applying the normal form theory and the center manifold argument for partial functional differential equations, the formula determining the properties of the Hopf bifurcation are obtained. Furthermore, the instability of the Hopf bifurcation leads to the emergence of spatial patterns. Finally, some numerical simulations are also carried out to illustrate and expand the theoretical results.

1. Introduction

It is well known that the dynamical behaviors (including stability, attractivity, persistence, periodic oscillation, bifurcation and chaos) of population models have become a subject of intense research activities. In particular, the properties of periodic solutions arising from the Hopf bifurcation are of great interest and many kinds of models such as predator–prey, neural network, epidemic disease and food chain have embodied these properties (see, for example, [1–6]). Predator–prey model is one of the most important population dynamical models. There are many factors which affect population dynamics in predator–prey models. One crucial component of predator–prey relationships is predator–prey interaction (also called functional response), which can be classified into many different types, such as Holling I–IV types, Hassell–Varley type, Beddington–DeAngelis type, Crowley–Martin type, and so on.

Recently, predator–prey interactions have been studied in structured populations in cyclical interactions with alliance-specific heterogeneous invasion rates in [7] and noise-guided evolution in [8], where it was shown that defensive alliances can emerge if the competing strategies are more than 3. Moreover, predator–prey interactions can emerge spontaneously in evolutionary settings relevant to public goods as reported in [9–11]. For more information on cyclical interactions, see a recently comprehensive review [12]. More recently, predator–prey interaction has been studied for a more elaborated social model, in which the individuals of one population gather together in herds, while the other one shows a more individualistic behavior [13]. Based on the fact that predator–prey interactions occur mainly through the perimeter of the herd, the authors in [13] have proposed a new predator–prey model described by the following ordinary differential equations

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$$\begin{cases} \frac{du}{dt} = u(1-u) - \sqrt{u}v, \\ \frac{dv}{dt} = \gamma v(-\beta + \sqrt{u}), \end{cases} \tag{1.1}$$

where $u(t)$ and $v(t)$ stand for the prey and predator densities, respectively, at time t. $\beta\gamma$ is the death rate of the predator in the absence of prey, γ is the conversion or consumption rate of prey to predator. This model is also known as the predator–prey model with herb behavior, and has been shown that the sustained limit cycles are possible and the solution behavior near the origin is more subtle and interesting than the classical predator–prey models, see [13,14].

Spatial diffusion is ubiquitous and can generate the rich spatiotemporal dynamics. The spatiotemporal dynamics of the predator–prey models involving spatial diffusion have been increasingly studied by many researchers. For instance, Hopf bifurcation, stationary pattern, Turing–Hopf bifurcation, Turing instability and pattern formation have been recently studied in [15–27]. Assuming that the prey and predator populations are in an isolate patch and neglecting the impact of migration, including immigration and emigration, and introducing the spatial diffusion into system (1.1), we have the following reaction–diffusion system

$$\begin{cases} \frac{\partial u}{\partial t} = u(1-u) - \sqrt{u}v + d_1\Delta u, \\ \frac{\partial v}{\partial t} = \gamma v(-\beta + \sqrt{u}) + d_2\Delta v, \\ u_x(0,t) = u_x(\pi,t) = v_x(0,t) = v_x(\pi,t) = 0, \quad t > 0, \\ u(x,0) = \phi(x) \geqslant 0, \quad v(x,0) = \psi(x) \geqslant 0, \quad x \in [0,\pi], \end{cases} \tag{1.2}$$

where d_1 and d_2 are the diffusion constants for the prey and predator, respectively.

In fact, the reproduction of predator after consuming the prey is not instantaneous, but is mediated by some time lag required for gestation. So, in this paper, we devote our attention to the following diffusive predator–prey model with herd behavior and delay

$$\begin{cases} \frac{\partial u}{\partial t} = u(1-u) - \sqrt{u}v + d_1\Delta u, \\ \frac{\partial v}{\partial t} = \gamma v(-\beta + \sqrt{u_t}) + d_2\Delta v, \\ u_x(0,t) = u_x(\pi,t) = v_x(0,t) = v_x(\pi,t) = 0, t \geqslant 0, \\ u(x,t) = \phi(x,t) \geqslant 0, \quad v(x,t) = \psi(x,t) \geqslant 0, \quad (x,t) \in [0,\pi] \times [-\tau, 0], \end{cases} \tag{1.3}$$

where the delay item $u_t = u(x, t-\tau)$ shows that the reproduction of predator after consuming the prey is not instantaneous, but mediated by some constant time lag τ for gestation. The effects of delay on population dynamics have been widely investigated, and are believed to be one major reason accounting for the nonlinear scenarios in population dynamics [28]. Meanwhile, many researchers have also concentrated on the reaction diffusion equations with delay and have obtained some interesting results such as delay-induced Hopf bifurcation [29–33], stability and global attractivity [34–36], pattern formation [37,38].

However, to the best of our knowledge, there are no results on the stability and Hopf bifurcations of the above system (1.3). In this paper, we concentrate our attention on the stability of the positive equilibrium, delay-induced Hopf bifurcation and the properties of Hopf bifurcation such as the direction of the bifurcation and stability of the bifurcating periodic solutions. Meanwhile, we find that the instability of the Hopf bifurcation, induced by diffusion and delay respectively, can lead to the emergence of spatial patterns. The rest of this paper is organized as follows. In Section 2, the stability of the positive equilibrium and the existence of Hopf bifurcations are investigated by analyzing the characteristic equations. In Section 3, the results of determining the direction and stability of the bifurcating periodic solutions are obtained. In Section 4, we illustrate our results with numerical simulations, which support and extend the theoretical results. The paper ends with a conclusion.

2. Stability of positive equilibrium and existence of Hopf bifurcations

In this section, we consider the stability of the positive equilibrium for system (1.3) and existence of Hopf bifurcations by analyzing the distribution of eigenvalues in corresponding linear system of system (1.3), which can be induced by diffusion and delay, respectively.

It is easy to check that system (1.3) has two boundary equilibria (0,0) and (1,0). And system (1.3) has a unique positive equilibrium (u^*, v^*) if and only if $0 < \beta < 1$, where

$$u^* = \beta^2, \quad v^* = \beta(1-\beta^2).$$

Without diffusion and delay, system (1.3) reduces into system (1.1) and the authors in [13] have obtained the following results for system (1.1) by the Poincaré–Bendixson theorem and the technique of phase plane analysis:

Lemma 2.1. *Assume* $0 < \beta < 1$ *and* $\beta_0 = \frac{\sqrt{3}}{3}$. *System* (1.1) *possesses a limit cycle and* (u^*, v^*) *is unstable if* $0 < \beta < \beta_0$; (u^*, v^*) *is globally asymptotically stable if* $\beta_0 < \beta < 1$; *system* (1.1) *undergoes a Hopf bifurcation near the positive equilibrium* (u^*, v^*) *if* $\beta = \beta_0$.

In the following, we study the effect of diffusion and delay on system (1.3). For simplification of notations, we always use $u(t)$ for $u(x,t)$, $v(t)$ for $v(x,t)$, $u(t-\tau)$ for $u(x, t-\tau)$, and $v(t-\tau)$ for $v(x, t-\tau)$. Let