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Oscillatory behavior of second-order half-linear damped dynamic equations

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ABSTRACT

By refining the generalized Riccati transformation technique, we obtain new oscillation criteria for a class of second-order half-linear delay dynamic equations with damping on a time scale. Assumptions in our theorems are less restrictive, these results complement and improve related contributions to the subject. In particular, the results obtained amend related results reported in the literature.

1. Introduction

The study of half-linear equations has become an important area of research due to the fact that such equations occur in a variety of real world problems such as in the study of p-Laplace equations, non-Newtonian fluid theory, and the turbulent flow of a polytrophic gas in a porous medium; see Agarwal et al. [3]. In this paper, we are concerned with oscillatory properties of a second-order half-linear dynamic equation

$$\left(a(x^{\Delta})^{\gamma}\right)^{\Delta}(t)+p(t)(x^{\Delta})^{\gamma}(t)+q(t)x^{\gamma}(\delta(t))=0, \tag{1.1}$$

where $t\in[t_0,\infty)_{\mathbb{T}}$, $\mathbb{T}$ is a time scale which is unbounded above. Throughout, we assume the following hypotheses:

(h_1) $\gamma\geqslant 1$ is a quotient of odd positive integers, $a,\ p,\ q:[t_0,\infty)_{\mathbb{T}}\to\mathbb{R}$ are rd-continuous functions satisfying $a(t)>0$, $a(t)-(\sigma(t)-t)p(t)>0$, and $q(t)>0$ for $t\in[t_0,\infty)_{\mathbb{T}}$;
(h_2) $\delta:[t_0,\infty)_{\mathbb{T}}\to\mathbb{T}$ is an rd-continuous function satisfying $\delta(t)\leqslant t$ and $\lim_{t\to\infty}\delta(t)=\infty$.

Since we are interested in oscillatory behavior of solutions near infinity, we assume that $\sup\mathbb{T}=\infty$ and define the time scale interval $[t_0,\infty)_{\mathbb{T}}$ by $[t_0,\infty)_{\mathbb{T}}:=[t_0,\infty)\cap\mathbb{T}$. By a solution of (1.1) we mean a nontrivial real-valued function x satisfying (1.1) for $t\in\mathbb{T}$. We recall that a solution x of (1.1) is said to be oscillatory on $[t_0,\infty)_{\mathbb{T}}$ if it is neither eventually positive nor eventually negative; otherwise, it is termed nonoscillatory. Eq. (1.1) is called oscillatory if all its solutions are oscillatory. Our attention is restricted to those solutions x of (1.1) which are not identically zero eventually.

For completeness, we recall the following concepts related to the notion of time scales. A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$. On any time scale we define the forward and backward jump operators by $\sigma(t):=\inf\{s\in\mathbb{T}|s>t\}$ and $\rho(t):=\sup\{s\in\mathbb{T}|s<t\}$, where $\inf\emptyset:=\sup\mathbb{T}$ and $\sup\emptyset:=\inf\mathbb{T}$, $\emptyset$ denotes the empty set. A point

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$t \in \mathbb{T}$ is said to be left-dense if $\rho(t) = t$ and $t > \inf \mathbb{T}$, right-dense if $\sigma(t) = t$ and $t < \sup \mathbb{T}$, left-scattered if $\rho(t) < t$, and right-scattered if $\sigma(t) > t$. Points that are right-scattered and left-scattered at the same time are called isolated. The graininess μ of the time scale is defined by $\mu(t) := \sigma(t) - t$, and for any function $f : \mathbb{T} \to \mathbb{R}$ the notation $f^\sigma(t) := f(\sigma(t))$. In order to prove the main results, we will use the chain rule formula

$$(f^\gamma)^\Delta(t) = \gamma f^\Delta(t) \int_0^1 \left[h f^\sigma(t) + (1-h) f(t)\right]^{\gamma-1} \mathrm{d}h, \quad \gamma \geqslant 1, \tag{1.2}$$

which is a simple consequence of Keller's chain rule (see Bohner and Peterson [9, Theorem 1.90]).

During the past decade, a great deal of interest in oscillatory and nonoscillatory behavior of different classes of dynamic equations on time scales has been shown, we refer the reader to [1,2,4–7,13–15,18,20–23,25,26]. In particular, oscillation of equations with damping has become an important area of research due to the fact that such equations arise in many real life problems; see, e.g., [8–12,16,24,27,28] and the references cited therein.

In the following, we briefly comment on related results that motivate our study. Agarwal et al. [2] and Şahiner [21] studied a second-order delay dynamic equation

$$x^{\Delta\Delta}(t) + p(t) f(x(\delta(t))) = 0.$$

Erbe et al. [13] established some oscillation results for

$$\left(r(t) x^\Delta(t)\right)^\Delta + p(t) f(x(\delta(t))) = 0 \tag{1.3}$$

under the conditions

$$\int_{t_0}^\infty \frac{\Delta t}{r(t)} = \infty$$

and

$$\int_{t_0}^\infty \frac{\Delta t}{r(t)} < \infty. \tag{1.4}$$

In particular, assuming (1.4) and

$$\int_{t_0}^\infty \frac{1}{r(t)} \int_{t_0}^t p(s) \Delta s \Delta t = \infty,$$

they presented a sufficient condition which ensures that the solution x of (1.3) is either oscillatory or satisfies $\lim_{t\to\infty} x(t) = 0$; see [13, Theorem 2]. Grace et al. [14] and Hassan [15] obtained several new oscillation criteria for a half-linear equation

$$\left(r (x^\Delta)^\gamma\right)^\Delta(t) + q(t) x^\gamma(t) = 0$$

in the cases

$$\int_{t_0}^\infty \frac{\Delta t}{r^{\frac{1}{\gamma}}(t)} = \infty$$

and

$$\int_{t_0}^\infty \left[\frac{1}{r(t)} \int_{t_0}^t q(s) \left(\int_s^\infty \frac{\Delta u}{r^{\frac{1}{\gamma}}(u)}\right)^\gamma \Delta s\right]^{\frac{1}{\gamma}} \Delta t = \infty. \tag{1.5}$$

For oscillation of damped equations on time scales, Bohner et al. [8] and Saker et al. [24] studied

$$x^{\Delta\Delta}(t) + q(t) x^{\Delta^\sigma}(t) + p(t) f(x^\sigma(t)) = 0.$$

Erbe et al. [12] investigated

$$\left(r (x^\Delta)^\gamma\right)^\Delta(t) + p(t) (x^{\Delta^\sigma})^\gamma(t) + q(t) f(x(\tau(t))) = 0,$$

and established new oscillation results provided

$$\int_{t_0}^\infty \frac{\Delta t}{R^{\frac{1}{\gamma}}(t)} = \infty$$

or

$$\int_{t_0}^\infty \left[\frac{1}{R(t)} \int_{t_0}^t Q(s) B^\gamma(s) \Delta s\right]^{\frac{1}{\gamma}} \Delta t = \infty, \tag{1.6}$$