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Linear long wave propagation over discontinuous submerged shallow water topography

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A R T I C L E I N F O

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A B S T R A C T

The dynamics of an isolated long wave passing over underwater obstacles are discussed in this paper within the framework of linear shallow water theory. Areas of practical application include coastal defense against tsunami inundation, harbor protection and erosion prevention with submerged breakwaters, and the construction and design of artificial reefs to use for recreational surfing. Three sea-floor configurations are considered: an underwater shelf, a flat sea-floor with a single obstacle, and a series of obstacles. A piecewise continuous coefficient is used to model the various sea-floor topographies. A simple and easily implementable numerical scheme using explicit finite difference methods is developed to solve the discontinuous partial differential equations. The numerical solutions are verified with the exact analytical solutions of linear wave propagation over an underwater shelf. The scope of this simplified approach is determined by comparison of its results to those of another numerical solution and wave transmission and reflection coefficients from experimental data available in the literature. The efficacy of approximating more complicated continuous underwater topographies by piecewise constant distributions is determined. As an application, a series of underwater obstacles is implemented.

1. Introduction

The study of water waves over variable underwater topography has numerous practical applications. Underwater reefs are built for several purposes including the generation of ideal waves for recreational surfing, protection from harbor damage and beach erosion, and the defense against destructive tsunami waves. Tsunamis are formed by rapid displacement of large masses of water, typically due to underwater earthquakes or volcanic activity [1,13]. As a tsunami wave approaches the shallow shorelines near a beach from deep water, its amplitude increases and its wavefront steepens; this is one source of its destructive potential. Studying the relationship between shallow water topography and the amplification of long waves that travel shore-wards can help us construct models that accurately predict the effects of tsunamis near coastal zones. These models can help us assess the efficacy of man-made structures in dampening the waves' destructive effects.

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For the purposes of evaluating breakwaters, the barriers' reflection and transmission coefficients are of fundamental importance. Lamb [12] first obtained the reflection and transmission amplitudes for a long wave incident at an underwater shelf. To extend Lamb's result to more realistic situations, many analytical solutions and numerical methods have since been developed. Analytical solutions have been obtained for piecewise linear topographies [2,10,16] and internal waves [21]. Various numerical approaches have used the eigenfunction expansion method for sinusoidal topographies [3], a series of obstacles [4], and permeable structures [17,18], Boussinesq equations [9,19,20], Boundary Element Method [5], nonlinear shallow-water equations [11], and Reynolds equations [15]. These methods, while having the advantage of giving accurate results when compared with experiments, are complicated to implement for a general type of wave profile. To solve more accurate equations such as the Reynolds equations for mean turbulent flow, much more complicated numerical solvers are needed. Even the analytical solutions obtained for linear flow would require multiple integral transforms and special functions to apply to tsunami-type isolated long waves. In practical applications, a coastal engineer, for example, only requires rough quantitative estimates to design effective breakwaters [24]. The loss in accuracy incurred by the study of simpler equations can be well compensated by the accessibility in implementation and the ease of obtaining results.

In this study, a numerical method is developed that can be easily implemented while also giving quantitative agreement with experimental data. The one-dimensional linear shallow water equations over discontinuous submerged topography are solved with finite difference methods. The problem of the discontinuous boundary conditions is approached with two types of solutions: introducing extra boundary conditions or applying a characteristic decomposition. The numerical solution is extended to a general case of arbitrary piecewise constant underwater topography. For special cases of an underwater shelf and obstacle, the numerical solution is compared with analytical solutions and reflection/transmission coefficients obtained experimentally and numerically available in the literature. Experimental data for a triangular underwater obstacle are compared with the numerical solution for a piecewise constant approximation to the triangular obstacle. The numerical solution is then applied to the approximation of more complicated underwater topographies (a series of obstacles and a submerged slope).

1.1. Governing equations

We first consider a model in which the topography of the ocean floor is generalized and then construct models with specific obstacle configurations. We introduce the relevant variables and physical constants in Table 1. Unless otherwise specified, we will use the subscript $_0$ to denote a characteristic dimensional quantity and the superscript $'$ to denote a dimensional variable.

A characteristic feature of a tsunami wave [7] is the smallness of the parameter $\frac{h_0}{\lambda} \ll 1$, where h_0 is the characteristic water depth of the ocean and λ is the wavelength. The presence of this small parameter allows us to use a system of partial differential equations (PDEs), namely the shallow water equations [22,25], for modeling tsunami wave dynamics. A detailed derivation of these equations from the general Navier–Stokes equations is given in [25]. If, in addition, the ratio of the characteristic wave amplitude to the water depth $\frac{\eta_0}{h_0} \ll 1$ is small, then the dynamics of such small amplitude long waves can be described by the linear shallow water equations [26]:

$$\frac{\partial u'}{\partial t'} + g\frac{\partial \eta'}{\partial x'} = 0, \quad \frac{\partial \eta'}{\partial t'} + \frac{\partial(\varphi'(x')u')}{\partial x'} = 0, \tag{1}$$

Table 1
Nomenclature table of variables and constants used.

Nomenclature	
η	Vertical elevation of water from quiescent position
u	Depth-averaged horizontal flow velocity
h	Distance from water surface to sea-floor
h_0	Dimensional water depth of sea-floor away from obstacle
h_i	Dimensional water depth over ith region
k_i	Dimensionless water depth over ith region; $k_i^2 = \frac{h_i}{h_0}$
x	Horizontal position coordinate
x_j	Horizontal position coordinate of jth region; $x_{j+1} = x_j + l_{j+1}$
l_j	Length of jth obstacle
t	Time variable
f	Initial waveform distribution
φ	Ocean floor topography
d	Distance of solitary wave peak from the $x = 0$ boundary
λ_0	Length of solitary wave
η_0	Characteristic wave amplitude
u_0	Characteristic fluid velocity
c_0	Characteristic velocity of wave propagation
λ_0	Characteristic length of initial wave
$t_0 = \frac{\lambda_0}{c_0}$	Characteristic time of propagation