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Compression of corneal maps of curvature

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ABSTRACT

We consider a map of curvature of the cornea that presents a central singularity. The application is that one of compressing the information of the map in order to recover it later from the selected information, in a faithful way and avoiding blurring effects.

1. Introduction

Starting from the vertex of the cornea (the point of the cornea closest to the device), the videokeratographer provides the local curvature along a meridian: it is the inverse of the calculated radius of the osculating circle.

The map with colored strips that is output of the videokeratographer gives the curvature of the cornea: precisely it shows us the values measured along a sequence of arcs, starting at the vertex, that accurately reconstruct a corneal profile along each of 256 radials. The radii of the arcs have been adjusted such that they reflect rays from the appropriate mires to the lens and produce the required ring pattern. It allows to put in evidence anomalies such as the keratoconus; it is used when planning a refractive surgical operation and when monitoring the healing afterwards [11].

A map from the videokeratographer Keratron 2000 requires $5 \times 10^5 - 1 \times 10^6$ bytes of storage in the bmp format. We compress the information of a map within the order of 10^3 bytes.

When recovering the map on the basis of the compressed information, the fast algorithm makes use of a tree data structure for the correct assignment of the color.

2. Map represented as strips; statement of the application

In Fig. 1 you see the map of curvature of the central part of a cornea represented as colored strips with colors from a palette $\mathcal{C} = \cup_{i=1}^{n(\mathcal{C})} \mathcal{C}_i$ where the white or the black are not present.

From a strip to the next one, in the verse from a colder to a warmer color, there is a constant step of decrease of the radius of curvature.

The directions of maximal and minimal curvatures at the vertex are placed on the image in white; the numbers in white are values of the radius of curvature, so the strips in warm red colors are relevant to the highest values of curvature.

One can notice the sharp transition in the values of the curvature around the vertex: there is strong toricity (astigmatism). There is a little keratoconus too; it is the region in red colors over the vertex.

The epithelium of the cornea is smooth. The exterior borders $\{\beta_i\}_i$ of the strips of curvature can be assumed to be smooth away from the vertex. This was the hypothesis also in [1]: there a Gaussian map was proposed to avoid the dependence on the location of the vertex, but it cannot detect toricities at the center of the pupil.

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The problem that we are concerned with is as follows: to provide a concise description of the map on the basis of which it is possible to make a good and fast reconstruction of the map itself.

It is not suitable to compute a 3*d* reconstruction because of the singularity. In fact a 3*d* reconstruction spanned by regular basis functions would present undue oscillations because of the Gibbs phenomenon; it would not be possible to make such oscillations small and to store few coefficients at the same time.

On the other hand an expansion in orthogonal Zernike polynomials with fixed number of terms can be inaccurate in abnormal conditions of the eye, see [12,7,13]; it cannot recover large localized curvatures of the severe keratoconus away from the vertex.

[5,9,10] proposed expansions alternative to the Zernike polynomials to recover the shape of the cornea from elevation data mainly. The surface data were fitted by splines or the approximation was found as a combination of functions from a dictionary. In [4] a mixed technique was proposed.

So the current approach is the one of reconstructing each exterior border of strip by its suitable concise description.

Now we state some notations. We denote a connected piece of exterior border by γ.

To each γ_i we associate its inner set that we denote $R_i = R(\gamma_i)$.

We use a data structure of the type tree to organize the inner sets $\{R_i\}_i$; such a tree is recalled in Section 3.

In Section 4 first we recall a simplification algorithm for γ and a subdivision algorithm to approximate a curve. These are ingredients for the current algorithm, presented in Section 5, that provides a concise description of γ.

Then in Section 6 we say about the computational cost and in Section 7 we say about the storage required to save the concise information; in Section 8 we show the examples.

3. Tree *T*: tool for colorization in reconstruction

In [8] we have described how to modify the values of the black or the white or almost white pixels, by assigning to each of them one color among those of the palette $\mathcal{C} := \{\mathbf{c}(l),\ l = 1, \ldots, n(\mathcal{C})\}$.

This preprocessing of the map is a necessary step before calculating the $\{\gamma_i\}_i$ that separate the strips with different colors of $\mathcal{C}$ (of course the directions of the principal curvatures and the values of radius, relevant to the pixels corrupted with white in the map, can be saved with negligible space of storage).

We compute the isolines relevant to the values of the color index at the pixels; the isoline points to the external border of the strip. Each one turns out to be a polygonal line joining a discrete set of knots. The locations of the knots are regularized a

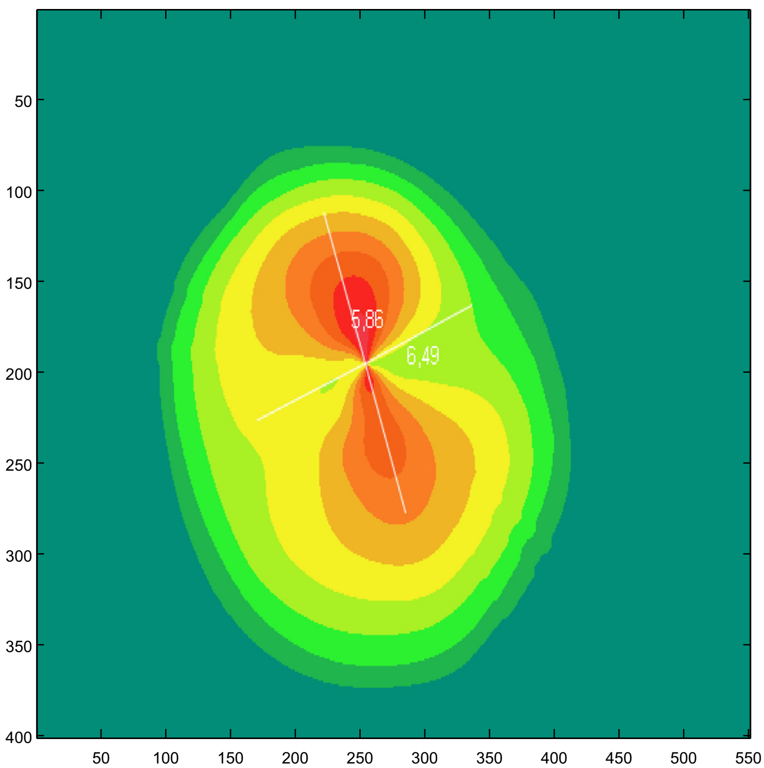

Fig. 1. Map of curvature.