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Generalized ordered weighted utility proportional averaging-hyperbolic absolute risk aversion operators and their applications to group decision-making

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ABSTRACT

This paper develops a new class of aggregation operator based on utility function, which introduces the risk attitude of decision makers (DMs) in the aggregation process. First, under the general framework of utility function, we provide a new operator called the generalized ordered weighted utility proportional averaging (GOWUPA) operator and study its properties which are suitable for any utility function. Then, under the hyperbolic absolute risk aversion (HARA) utility function, we propose another new operator named as the generalized ordered weighted utility proportional averaging-hyperbolic absolute risk aversion (GOWUPA-HARA) operator, and further investigate its families including a wide range of aggregation operators. To determine the GOWUPA-HARA operator weights, we put forward an orness measure of the GOWUPA-HARA operator and analyze its properties. Considering that different DMs may have different opinions towards decision-making and their opinions can be characterized by different orness measures, we construct a new nonlinear optimization model to determine the optimal weights which can aggregate all the individual sets of weights into an overall set of weights. Furthermore, based on the GOWUPA-HARA operator, a method for multiple attribute group decision- making (MAGDM) is developed. Finally, an example is given to illustrate the application of the GOWUPA-HARA operator to the MAGDM.

1. Introduction

Multiple attribute group decision making (MAGDM) is an important research topic, which determines the ranking order of the alternatives to help decision makers (DMs) to make their decisions [1]. To choose a desirable alternative, DMs often present their preference information which needs to be aggregated via some proper approaches. There are many methods for aggregating information [2–42]. One of the most popular methods for aggregating decision formation is the ordered weighted averaging (OWA) operator developed by Yager [26]. It provides a general class of parametric aggregation operators and has been proved to be useful for studying many different kinds of aggregation problems. Up to now, OWA operator has been used in a wide range of applications [11,28].

Motivated by the OWA operator, an extension of the OWA operator is the generalized OWA (GOWA) operator, which combines the OWA operator with the generalized mean operator [27]. It generalizes a wide range of aggregation

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operators such as the OWA operator, the ordered weighted geometric averaging (OWGA) operator [24], the ordered weighted harmonic averaging (OWHA) operator [27]. Based on an optimization theory, Zhou and Chen [32] presented a generalized ordered weighted proportional averaging (GOWPA) operator, which is an extension of the OWGA operator. Additionally, a generalized ordered weighted exponential proportional aggregation (GOWEPA) operators and a generalized ordered weighted multiple averaging (GOWMA) operator were also developed by Zhou et al. [33,34]. Other extension of the OWA operator can be found in literature [8,20–22,30]. The above aggregation operators, however, only focus on using the mean to eliminate the difference of variable values and do not consider DMs' risk attitude in the aggregation process.

Another key issue of applying the OWA operator for MAGDM is how to determine the associated weights. Many researchers have focused on this point and developed some useful approaches to obtaining the OWA weights. For instance, O'Hagan [13] suggested a maximum entropy approach for obtaining the OWA operator weights under a given orness measure. Wang and Parkan [17] proposed a minimax disparity approach for obtaining OWA operator weights under a given orness measure. Liu and Chen [7] put forward a parametric geometric method that could be used to obtain the maximum entropy weights. Fullér and Majlender [5] proposed a minimum variance method to obtain the OWA operator weights based on a given orness measure. Wang et al. [43] constructed the least squares deviation (LSD) model for determining the OWA operator weights under a given orness measure. Other approaches to determining the OWA operator weights can be found in literature [8,21,22,44–48,25,49–52]. The approaches mentioned above assume that any individual weighting vector is equal to the optimal weighting vector, and correspondingly there exists only one orness measure to characterize DMs' attitude towards decision-making. As a result, there is only one set of OWA operator weights to be generated. However, this is not consistent with the real situation. In fact, multiple DMs may join in decision- making process to reach a holistic opinion that reflects a collective view of all the participants. Different DMs may have different orness measures in the decision-making process, and then the corresponding OWA operator weights may also be different. So it is necessary to develop a new method to aggregate the preferences of all participants in MAGDM.

The aim of the paper is to develop a new class of aggregation operator based on utility function, which incorporates the risk attitude of DMs in the aggregation process. Under the general framework of utility function and based on an optimal deviation model, we firstly provide a new operator called the generalized ordered weighted utility proportional averaging (GOWUPA) operator, and then by studying its properties we find that it is commutative, monotonic, bounded and idempotent. These properties are suitable for any utility function. Furthermore, we concentrate on the hyperbolic absolute risk aversion (HARA) utility function, which is rather rich, e.g., by suitable adjustment of the parameters we can respectively obtain power utility function and exponential utility function. Under the HARA utility, we propose another new operator called the generalized ordered weighted utility proportional averaging-hyperbolic absolute risk aversion (GOWUPA-HARA) operator, and study its families which include a wide range of aggregation operators such as the OWGA operator, OWPA operator, OWHPA operator, GOWPA operator, maximum operator, minimum operator. The main advantage of the GOWUPA-HARA operator is that it can not only reflect the DMs' risk attitude towards the aggregation information, but also provide a very general formula including a wide range of aggregation operators.

To determine the GOWUPA-HARA operator weights, we provide an orness measure of the GOWUPA-HARA operator, which is an extension of the orness measure of the GOWPA operator derived by Zhou and Chen [32]. We further investigate some properties associated with this orness measure. Considering that different DMs may have different perspectives towards decision- making, which can be characterized by different orness measures, this situation results in different sets of the GOWUPA-HARA operator weights corresponding to different orness measures, we construct a new nonlinear weighting model to determine the optimal weights of the GOWUPA-HARA operator which can aggregate all the individual sets of weights into an overall set of weights. The main advantage of the weighting model is that it can not only minimize the differences between the orness measures provided by each DM and the orness measure corresponding to an optimal weighting vector, but also produce as equally important weights as possible.

Furthermore, based on the GOWUPA-HARA operator, a new approach for MAGDM is developed, which can also be used to solve different group decision-making problems such as assessment of energy investment alternatives and evaluation of manufacturing system investments. Finally, an example of the investment selection is provided to examine our approach.

The rest of the paper is organized as follows. Section 2 reviews the OWA, OWGA and GOWPA operators and introduces an HARA utility function. Section 3 presents a GOWUPA operator and analyzes its properties. Especially, in this section we provide a GOWUPA-HARA operator and identify its families. Section 4 proposes an orness measure of the GOWUPA-HARA operator and discusses its properties. In particular, we further provide a nonlinear model for determining the optimal weights which can aggregate each DM's opinion. Section 5 develops an approach for MAGDM under GOWUPA-HARA operator. An illustrative example is provided in Section 6 and the conclusions are drawn in Section 7.

2. Preliminaries

This section briefly reviews the OWA, OWGA and GOWPA operators and introduces an HARA utility function which later will be used to develop a new aggregation operator in this paper.