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The dynamics of a time delayed epidemic model on a population with birth pulse ☆

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ABSTRACT

An epidemic model with time delay and impulse is proposed to describe if time delay plays an important role in the spread of an infectious disease. Mathematical analyses with regard to the local, global stability of equilibrium are performed. We give out the conditions for global attractivity of the disease-free equilibrium and permanence of the system, respectively.

1. Introduction

It is well known that epidemiology is a major field of research, there are many epidemiological models [1–12]. In many epidemic models, the total constant population is divided into four classes: susceptible S, exposed E, infectious I, removed R. These models are called *SEIRS*, since susceptible become exposed, then infectious, then removed with temporary immunity and then susceptible again. In this paper, we suppose that the disease can hardly been cleared, the susceptible become exposed, then infectious, then removed into the exposed after treatment. Moreover, the incidence of a disease is the number of new cases per unit time and plays an important role in the study of mathematical epidemiology. The general form of incidence is written as βSI, S and I are the numbers of susceptible and infective individual at time t, respectively, β is the probability per unit time of transmitting the infection between two individuals taking part in a contact. There also exists the incidence forming βS^pI^q in some models [17]. They formed the values of p and q can influence the number of equilibrium and the dynamical behavior. In addition, a model with a general nonlinear incidence βS^pI^q is studied in [18]. In the following, we choose the parameter p of the incidence βS^pI^q as $p = 1$. When $q = 1$, the incidence βSI^q is just as βSI.

In recent years, impulsive systems are found in many domains of applied sciences (see [13–16,21]). In [21], Li et al. considered a prey-dependent model with impulsive state feedback control. There are many studies in impulsive differential systems with impulses at fixed moments $t = t_k$ or even periodic fixed moments $t = nT$, such as vaccination [22–25], chemotherapeutic treatment of disease [26,27] and birth pulse [28–30]. In [30], Liu and Chen studied the existence of positive periodic solution of two-species competitive system with toxicant and birth pulse by using the method of coincidence degree. The authors discussed the properties of its solutions of the system, such as, the existence, uniqueness, stability and periodicity, etc.

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There are many papers considering time delays in epidemic models [18–20]. In [18], Hethcote et al. considered the dynamics of the system with the time delay, which is the immunity period of the removed class. In [19], two time delays are introduced and studied in an *SEIRS* model, where the two delays represent the latent and immune periods, respectively. They presented local stability analysis of equilibrium and obtained sufficient conditions for global stability of disease-free equilibrium. By neglecting disease-related death rates in the *SEIRS* model in [19], Wang [20] shown the stability of equilibria and the uniformly persistence in the population. In this paper, we establish a new *SEIE* epidemic model including a general incidence forming βSI^q. Our goal is to determine the impact of the latent period and impulse birth on dynamics of the *SEIE* model.

Our paper is organized as follows. In Section 2 we formulate the complete mathematical model and give its biological interpretation and preliminaries. The global attractivity of the disease-free periodic solution is discussed in Section 3. Section 4 is devoted to the permanence of the model with the time delay and impulse birth. Section 5 is the conclusion of the paper.

2. Model and preliminaries

For understanding the spread and control of infectious diseases with the different infectious rate, various continuous epidemic models have been studied (for example, [16]). In this paper, we suppose $\beta S(t)I^q(t)$ is the infectious rate, where $q \in R$ is the mutual interference coefficient. The *SEIE* epidemic model with time delay is the following nonlinear ordinary differential system:

$$\begin{cases} \frac{dS(t)}{dt} = \alpha - \mu S(t) - \beta S(t) I^q(t), \\ \frac{dE(t)}{dt} = \lambda\beta S(t) I^q(t) - \lambda\beta e^{-\mu\tau} S(t-\tau) I^q(t-\tau) - \mu E(t) + \gamma I(t), \\ \frac{dI(t)}{dt} = \lambda\beta e^{-\mu\tau} S(t-\tau) I^q(t-\tau) - (\mu_1 + \gamma) I(t) - \rho I^2(t), \end{cases} \tag{2.1}$$

where $S(t)$, $E(t)$ and $I(t)$ are susceptible, latent and infective classes at time t, respectively. The constant $\alpha > 0$ represents the immigration rate, assuming all newborns to be susceptible. The constant $\mu > 0$ represents the natural death rate of the susceptible and the latent classes. The constant $\mu_1 > 0$ represents the death rate related to the disease of the infective class. It is natural to assume biologically that $\mu_1 > \mu$. The constant β is contact rate and γ is the recovery rate from the infective class to the latent class. ρ is the density dependent rate of the infective individuals. τ is the period of latent of disease.

Recently, there exist continuous epidemic model with pulse birth. Pulse birth is defined as the repeated application of birth at discrete time with equal interval.

Considering the pulse birth in the above continuous *SEIE* epidemic model (2.1), we can construct the following impulsive differential system:

$$\begin{cases} \left.\begin{aligned} &\frac{dS(t)}{dt} = \alpha - \mu S(t) - \beta S(t) I^q(t) \\ &\frac{dE(t)}{dt} = \lambda\beta S(t) I^q(t) - \lambda\beta e^{-\mu\tau} S(t-\tau) I^q(t-\tau) - \mu E(t) + \gamma I(t) \\ &\frac{dI(t)}{dt} = \lambda\beta e^{-\mu\tau} S(t-\tau) I^q(t-\tau) - (\mu_1 + \gamma) I(t) - \rho I^2(t) \end{aligned}\right\} \quad t \neq nT, \\ S(t^+) = S(t) + \xi, \quad t = nT,\ n \in Z^+, \end{cases} \tag{2.2}$$

where the parameter ξ is the proportion of birth of susceptible population, T is a fixed positive constant and denotes the period of the impulsive effect, and $T = 0, 1, 2, \ldots$

The initial conditions for system (2.2) are

$$(\phi_1(s), \phi_2(s), \phi_3(s)) \in C([-\tau, 0], R^3_+), \quad \phi_i(0) > 0, \quad i = 1, 2, 3,$$

$$R^3_+ = \{x \in R^3 : x \geqslant 0\}.$$

From the point of biology, we only consider system (2.2) in the biological meaning region: $D = \{(S, E, I) | S, E, I \geqslant 0\}$.

Note that the variables E does not appear in the first and third equations of system (2.2), hence we will consider the subsystem of (2.2) as follows:

$$\begin{cases} \left.\begin{aligned} &\frac{dS(t)}{dt} = \alpha - \mu S(t) - \beta S(t) I^q(t) \\ &\frac{dI(t)}{dt} = \lambda\beta e^{-\mu\tau} S(t-\tau) I^q(t-\tau) - (\mu_1 + \gamma) I(t) - \rho I^2(t) \end{aligned}\right\} \quad t \neq nT, \\ S(t^+) = S(t) + \xi, \quad t = nT,\ n \in Z^+. \end{cases} \tag{2.3}$$

In the following, we mainly consider the global attractivity of infection free periodic solution and the uniform permanence of system (2.3), before introducing our main results, we give some preliminaries needed in next section.

Denote by $f = (f_1, f_2, f_3)^T$ the map defined by the right-hand sides of the first three equations of system (2.2).

Let $V : R_+ \times R^3_+ \longrightarrow R_+$, then V is said to belong to class V_0 if:

(i) V is continuous in $((n-1)T, nT] \times R^3_+$ and for each $x \in R^3_+$,

$$\lim_{(t,s) \longrightarrow ((n-1)T^+, z)} V(t, s) = V((n-1)T, z)$$