



Zero-dissipative semi-implicit hybrid method for solving oscillatory or periodic problems



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ABSTRACT

In this paper, a new semi-implicit two-step hybrid method with fifth algebraic order is derived for the integration of second-order oscillatory initial value problems. The new method possesses dispersion of order eight and dissipation of order infinity. Numerical experiment reveals the superiority of the new method for solving oscillatory or periodic problems over several methods of the same algebraic order in the literature.

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1. Introduction

In the last few decades, there has been growing interest in the research of new numerical techniques for approximating the solution of second order initial value problem

$$y''(x) = f(x, y), \quad y(x_0) = y_0, \quad y'(x_0) = y'_0, \quad (1)$$

which is independent on y' explicitly. This type of problem arises in different fields of science and engineering, which includes quantum mechanics, celestial mechanics, molecular dynamics, quantum chemistry, astrophysics, electronics, semi-discretizations of wave equation, and so on.

Due to their importance, many numerical methods have been derived for approximating their solutions, some of which are Runge–Kutta methods, Runge–Kutta Nyström, linear multi step methods and so on. For the Runge–Kutta methods and other related methods specifically derived for approximating the solutions of first order IVPs, the second order IVPs need to be transformed into a system of first order IVPs so that the methods can be applied. In the quest for methods that best approximate the solutions of (1) many authors considered different modifications on Runge–Kutta methods, multistep methods and Runge–Kutta Nyström methods, [2–4,7–13,15,16,18–20]. Hybrid type methods related to multistep methods are proposed by many authors for approximating the solutions of (1), for example, see [21,23,22,5]. But most of the multi step hybrid methods are characterized by off-step points and higher stages, which make them expensive. Hence, Franco [10] identified the drawback of most of the multistep hybrid methods as regards to high computation cost, and therefore proposed explicit two-step hybrid methods up to algebraic order six with less computation cost by using the algebraic order conditions of two-step hybrid methods developed in [6]. In furtherance to this, Ahmad et al. [1] proposed semi-implicit hybrid methods up to algebraic order five. In this paper, we derive a semi-implicit hybrid method with improved numerical properties. The method is generally given by

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$$Y_i = (1 + c_i)y_n - c_i y_{n-1} + h^2 \sum_{j=1}^i a_{ij} f(x_n + c_j h, Y_j),$$

$$y_{n+1} = 2y_n - y_{n-1} + h^2 \sum_{i=1}^m b_i f(x_n + c_i h, Y_i), \quad i = 1, \dots, m, \tag{2}$$

where y_{n+1} and y_{n-1} are approximations to $y(x_{n+1})$ and $y(x_{n-1})$ respectively. The parameters c_i, b_i and a_{ij} are assumed to be real. Let \mathbf{b} and \mathbf{c} be m -dimensional vectors given by $\mathbf{b} = [b_1, b_2, b_3, \dots, b_m]^T$ and $\mathbf{c} = [c_1, c_2, c_3, \dots, c_m]^T$ and $\mathbf{A} = [a_{ij}]$ be an $m \times m$ matrix, then the coefficients of the methods can be summarized as in Table 1. In Section 2, we discuss dispersion and dissipation of the method and their conditions are derived. Derivation of the method is fully presented in Section 3. Stability analysis of the method is presented in Section 4. Numerical results are given in Section 5. And finally, some conclusions are presented in Section 6.

2. Dispersion and dissipation analysis

In this section we discuss dispersion and dissipation of semi-implicit hybrid methods. Consider the homogeneous test problem

$$y''(x) = -\alpha^2 y(x), \quad \alpha > 0, \quad \alpha \in R. \tag{3}$$

Applying (2) on the test Eq. (3) we obtain the following equations in vector form

$$\mathbf{Y} = (\mathbf{e} + \mathbf{c})y_n - \mathbf{c}y_{n-1} - z^2 \mathbf{A}\mathbf{Y}, \tag{4}$$

$$y_{n+1} = 2y_n - y_{n-1} - z^2 \mathbf{b}^T \mathbf{Y}, \tag{5}$$

where $z = \alpha h$, $\mathbf{e} = [1, 1, 1, \dots, 1]^T$, vectors \mathbf{b}, \mathbf{c} and matrix \mathbf{A} are given in Section 1. From (4) we obtain

$$\mathbf{Y} = (\mathbf{I} + z^2 \mathbf{A})^{-1} (\mathbf{e} + \mathbf{c})y_n - (\mathbf{I} + z^2 \mathbf{A})^{-1} \mathbf{c}y_{n-1}, \tag{6}$$

where \mathbf{I} is identity matrix and vector $\mathbf{Y} = [Y_1, Y_2, Y_3, \dots, Y_m]$. Substituting (6) into (5) we obtain

$$y_{n+1} - T(z^2)y_n + D(z^2)y_{n-1} = 0, \tag{7}$$

where

$$T(z^2) = \frac{2 + \sum_{i=1}^m U_i z^{2i}}{\prod_{i=1}^m (1 + z^2 a_{ii})}, \quad D(z^2) = \frac{1 + \sum_{i=1}^m V_i z^{2i}}{\prod_{i=1}^m (1 + z^2 a_{ii})}, \tag{8}$$

and U_i, V_i , depend on a_{ij}, b_i, c_i .

The solution of the difference Eq. (7), [24] is given by

$$y_n = 2|c||\rho|^n \cos(\omega + n\phi), \tag{9}$$

where ϕ is the phase, ρ is the amplification factor, c and ω are real constants determined by y_0 and y'_0 and the parameters of the method. The solution of the test problem (3) is

$$y(x_n) = 2|\delta| \cos(\psi + nz), \tag{10}$$

where δ and ψ are real constants determined by initial conditions and n is the number of terms. The definition formulated by Van der Houwen and Sommeijer [24] follows immediately.

Definition 1. Apply the hybrid method (2) and (3). The quantity $R(z) = z - \phi$ is called phase lag or dispersion error of the method. The method is said to have dispersion error of order q if $R(z) = O(z^{q+1})$. Furthermore, the quantity $S(z) = 1 - |\rho|$ is called dissipation error of the method. And the method is said to be dissipative of order r if $S(z) = O(z^{r+1})$.

From Definition 1, it follows that

$$R(z) = z - \cos^{-1} \left(\frac{T(z^2)}{2\sqrt{D(z^2)}} \right), \tag{11}$$

$$S(z) = 1 - \sqrt{D(z^2)}. \tag{12}$$

Table 1
Summary of coefficient.

\mathbf{c}	\mathbf{A}
	\mathbf{b}^T

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